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VOLUME II - PART I
PROBLEM DEFINITION

Compilation Report
for
ADVANCED SPACEBORNE DETECTION, TRACKING, AND NAVIGATION SYSTEMS STUDY AND ANALYSIS

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ABSTRACT

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This document is Volume II, the Problem Definition, of a five-volume report compiled for the Marshall Space Flight Center by the Aerospace Division, Westinghouse Defense and Space Center, Baltimore, from industry studies conducted for the purpose of consolidating and extending studies of detection, tracking navigation and guidance systems for future space missions.

Volume II establishes the nature, relative importance, and potential growth of missions to be attempted in the near future and provides a Problem Definition to serve as the basis for the Analytical Solution of Volume III

In this volume, the general problem of determining system requirements for future space missions is developed into the specific problem of the determination of navigation and control sensor requirements for local or onboard guidance of the manned or unmanned lunar mission. As a preliminary to analysis, the lunar missions is divided into phases. Those considered for analysis are, in order of mission occurrence; Midcourse Phase, Parking Orbit and Descent Phase, Lunar Landing Phase, Lunar Ascent Phase, and Lunar Rendezvous Phase. Trajectory and guidance models and an analytical plan are developed for each phase for the subsequent analyses conducted in Volume III. Both the Problem Definition and the Analytical Solution for Earth Orbital Rendezvous are contained in Volume IV.

These studies have been organized along guidelines furnished by MIL-D-8684A, paragraphs 3.4.3.1 and 3.4.3.2.

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DEFINITION OF TERMS

Listed below are some common terms which are used often in this report. The definitions pertain to the usage of these words in this report and are not necessarily intended to be general.

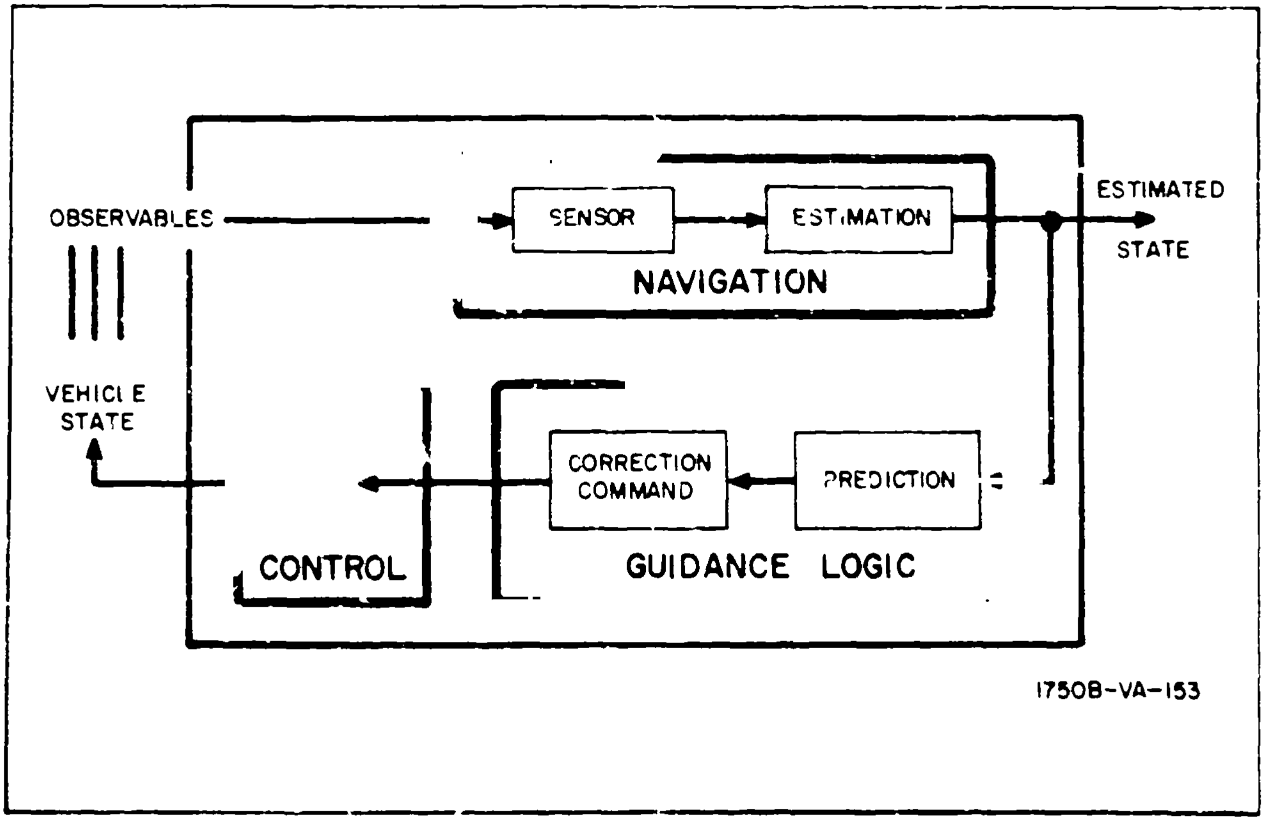
<u>mission</u> - a particular type of space flight; e.g., manned lunar mission, unmanned Mars mission, etc.

<u>phase</u> - some time-segment of a space mission which is significantly different from other time segments; e.g., mid-course phase, landing phase, etc.

<u>state</u> - the complete specification of a space vehicle's translational location and motion; i.e., three components each of position and velocity at a particular time.

<u>observable</u> - an observable is some measurable quantity which is related to the vehicle state, such as range or range rate.

<u>guidance</u> - a generic term covering the overall problem of causing the space vehicle to arrive at some desired target location; includes Navigation, Guidance Logic, and Control (see below).



navigation - the subsystem of the guidance system in which the estimated state of the vehicle is developed. Includes sensing (of observables) and estimation.

guidance logic - the subsystem of the guidance system in which the estimated state is used to compute some steering or correction commands which will cause the spacecraft to arrive at its desired destination.

control - the subsystem of the guidance system in which control commands are implemented with thrust control.

trajectory - the time history of spacecraft position and velocity.

nominal trajectory - precomputed trajectory which is defined as the trajectory which will be flown if no guidance system errors occur.

SUMMARY

The Problem Definition presented in this volume has as its objective to provide a clear understanding of the problem to be solved and consists of a general and a specific definition of the problem requirements derived from an overall examination of the problem. Major categories requiring explicit definition include the nature and priority rating of anticipated space missions and the pertinent characteristics of trajectories, guidance laws, and systems relating to NASA's future space goals.

In Section 1 a brief history of the study program and its principal objectives are given and the study methods used to perform the studies are described.

In Section 2 the various space missions and mission phases required to define the scope of the problem are discussed. An order of mission priority is established as follows:

1. Manned Lunar Mission

2. Earth Orbital Rendezvous

3. Unmanned Lunar Mission

4. Manned Interplanetary Mission

The lunar mission is then divided into mission phases for purposes of analysis and a typical mission profile, similar to the Apollo mission profile, is chosen for analysis of the manned lunar mission.

In Section 3 a general statement of the guidance problems to be solved are given along with the Part II study objectives. Emphasis on navigation sensor accuracy and limitation of the study to onboard systems is discussed.

In Sections 4 through 8 the Lunar Midcourse, Lunar Parking Orbit and Descent, Lunar Landing, Lunar Ascent, and Lunar Rendezvous Phases of the manned lunar mission are examined, and in these sections trajectory and guidance system models are developed preparatory to the Part II analysis of Volume III.

1. INTRODUCTION

The continuing rapid advancement of United States space capability will depend upon increasing present state of the art technical capabilities in many areas. One of these areas, which will assume greater importance as more complicated and more distant space missions are attempted, is guidance of spacecraft. Most space flights to date have been relatively simple from a guidance standpoint for two reasons:

a. Ground control of all operations has been feasible due to the relative closeness and visibility of the space vehicle.

b. Guidance operations have been theoretically straightforward in most cases, since the vehicle is in ballistic flight after burnout of the orbital booster.

Although there have been some exceptions to the above restrictions, such as Syncom and Mariner, guidance operations on past space missions have been relatively simple compared to guidance requirements on the Apollo mission. Thus, it may be said that the Apollo mission will usher in a whole new era in space guidance since operations such as thrusting into a lunar orbit, lunar orbit determination, lunar landing, lunar ascent, rendezvous, and return to earth will be required. These operations will be typical of lunar and planetary missions in the post-Apollo period.

Over the past few years a large amount of literature has been generated by various investigators in the areas of lunar and interplanetary flight. While it might have been expected that as a result of the many extensive spaceflight research programs being performed many of the performance requirements for guidance system sensors would have been analytically derived, verified, and published, in many areas this has not been the case. Many such studies either analyze some small segment of a mission for a special set of conditious and without regard for the overall mission, or else are performed with the objective of demonstrating that a given sensor performance is sufficient for some mission segment without examination of what performance is necessary and without regard for what sensor configurations are optimum. Nevertheless, much of this material is useful to serve as a basis for the objective determination of sensor requirements in areas where this has not previously been done.

In July of 1962, parallel contracts of seven months duration were issued by the NASA Headquarters Office of Advanced Research and Technology, to three contractors: Cornell Aeronautical Laboratory, Inc., Raytheon Co. Missile and Space Division, and the Aerospace Division of the Westinghouse Defense and Space Center. The common objective of these studies was "To establish requirements for advanced spaceborne detection, tracking, and navigation systems employed in manned and unmanned space missions". Specific tasks were:

a. Conduct a survey of available pertinent information and literature and, on this basis generate a Problem Definition (Part I Engineering Report) which establishes the nature, relative importance, and growth potential of missions considered for the prescribed period and which formulates an analytical study program for the objective determination of sensor performance requirements in areas where this has not previously been done.

b. Using the Problem Definition as an input, generate an Analytical Solution (Part II Engineering Report) to determine sensor requirements on the basis of required system performance and consider these in terms of present state of the art, for various sensor types.

The contracts stated that paragraphs 3. 4. 3. 1 and 3. 4. 3. 2 of MIL-D-8684A (Aer), paragraph 3. 4. 3, were to be used as a guideline for conducting the studies.

Technical cognizance at contract inception was vested with the NASA Langley Research Center. Technical cognizance of the last three months of the contract duration was given to the Marshall Space Flight Center of NASA.

This compilation report, Volume II, integrates the Part I efforts of the three contractors into a Part I Problem Definition for this compilation report set of documents.

1.1 DAMV METHOD

The study method of MIL-D-8684A(Aer), paragraph 3. 4. 3, is often referred to as the DAMV (Definition, Analysis, Mechanization, Verification) method and is frequently used in the design of complex weapons systems for the Navy. (Ref. 1-1) A block diagram of the method is shown in figure 1-1.

In Part I, Problem Definition, the operational requirements and the fixed problem constraints are utilized as inputs to develop system concepts and mathematical models for analysis of requirements. The Part I outputs are inputs to Part II, Analytical Solution, in which mathematical analysis is performed on the system concepts, utilizing the analytical models developed in Part I. The result of this analysis is the set of functional requirements and constraints which will solve the model problem generated in Part I. Note that

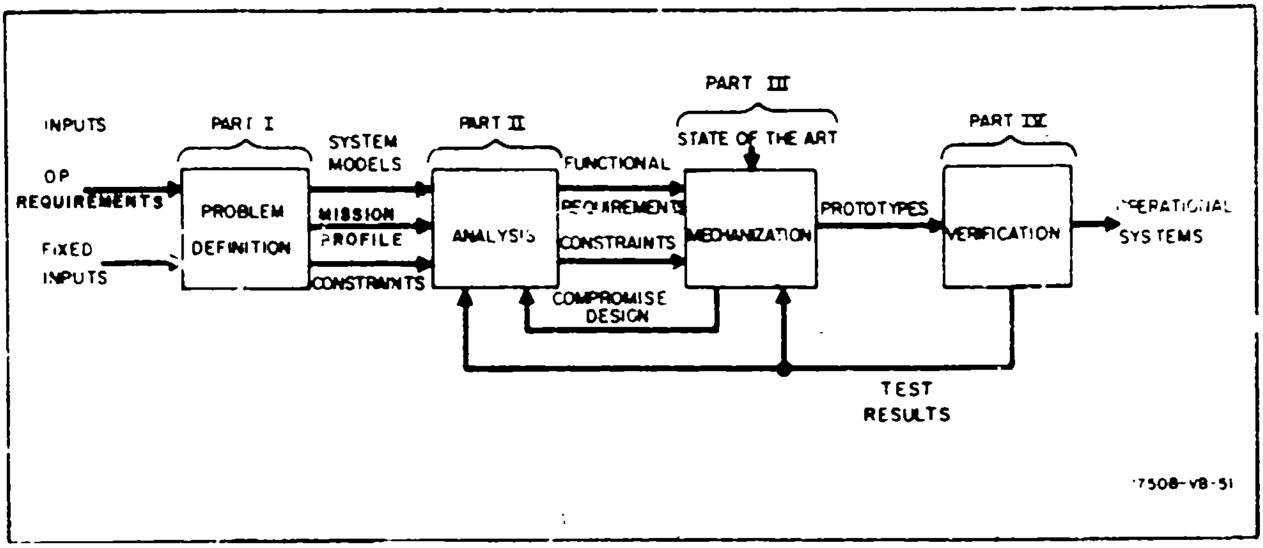


Figure 1-1. Block Diagram of the DAMV Method of Systems Development

although state of the art in equipment development may be considered in Part II, it is not one of the primary inputs and does not act as a rigid constraint on functional requirements. This notion is the key idea in the whole DAMV approach; i.e., functional requirements are developed somewhat apart from specific hardware considerations, instead of determining the performance obtainable from a specific set of equipment.

Two advantages of this method are apparent:

a. Requirements are limited only by considerations of the basic physics of the problem.
b. The comparison between different equipments to perform the same job becomes more readily apparent.

In Part III, Mechanization, explicit system design and fabrication is undertaken and, accordingly, state of the art hardware limitations play a direct role. Compromises of functional requirements and hardware limitations are evaluated in order to arrive at a reasonable design. When prototype equipment has been designed and developed, Part IV, Verification (equipment testing), is begun. Then deficiencies in system design which show up in testing are fed back into Part II in order to analyze the effect on overall performance of a system that is not ideal.

The DAMV method just described is for the complete design and development of a system from original concept to operational use. However, since the NASA contracts were for paper studies rather than operational equipment, only the first two parts of the process, Problem Definition and Analytical Solution, were employed. A more detailed outline of how these phases are utilized for the problem of interest is given in the following subsection.

1.2 OUTLINE OF STUDY EFFORT

Figure 1-2 is a detailed block diagram of the first two parts of the DAMV approach as they are actually applied to this study.

1.2.1 Problem Definition

In Figure 1-2, the upper line is Part I, the Problem Definition, which is described in this paragraph. The first step was to clarify and define the scope of the work to be done to ensure agreement with the contracting agency on the problems to be investigated. This was accomplished by surveying the important literature pertaining to spaceflight and by trips to NASA centers to determine future spaceflight plans by making a preliminary examination of some of the missions. This examination of missions revealed that although there are a great many possible space missions, most of these have well-defined phases (Launch, Midcourse, Rendezvous, etc.) which are similar in principle; i.e., the difference between phases of any mission is much greater than the difference between similar phases of different missions. As a result, it was decided to subdivide the analytical work according to phases rather than missions. The delineation of mission phases, together with accounts of trips to NASA centers, was presented to NASA Langley in a preliminary work statement and approved. Work was then begun on the detailed Problem Definition.

The work done in the Problem Definition included selection of mission priorities, delineation of the phases to be analyzed, the development of assumptions and constraints on the problem, the listing of expected environmental conditions, and the generation of an analytical plan.

Finally, the system models and trajectory models for analysis were chosen for each of the five phases of the Manned Lunar Mission. These models served as inputs to Part II, the Analytical Solution.

Mission priority was assigned to various missions using criteria such as mission importance, guidance operations required, and mission probability. The Manned Lunar Mission was selected as being of highest priority in the post-Apollo era, with three other missions assumed to be of lesser priority.

It was decided to extensively analyze five space-flight phases: Midcourse, Lunar Orbital, Lunar Landing, Lunar Ascent, and Rendezvous. It was felt that this selection effectively covers the spectrum of difficult guidance tasks, except for earth launch and reentry. These latter two phases were deleted by NASA Langley primarily because they have been extensively analyzed elsewhere.

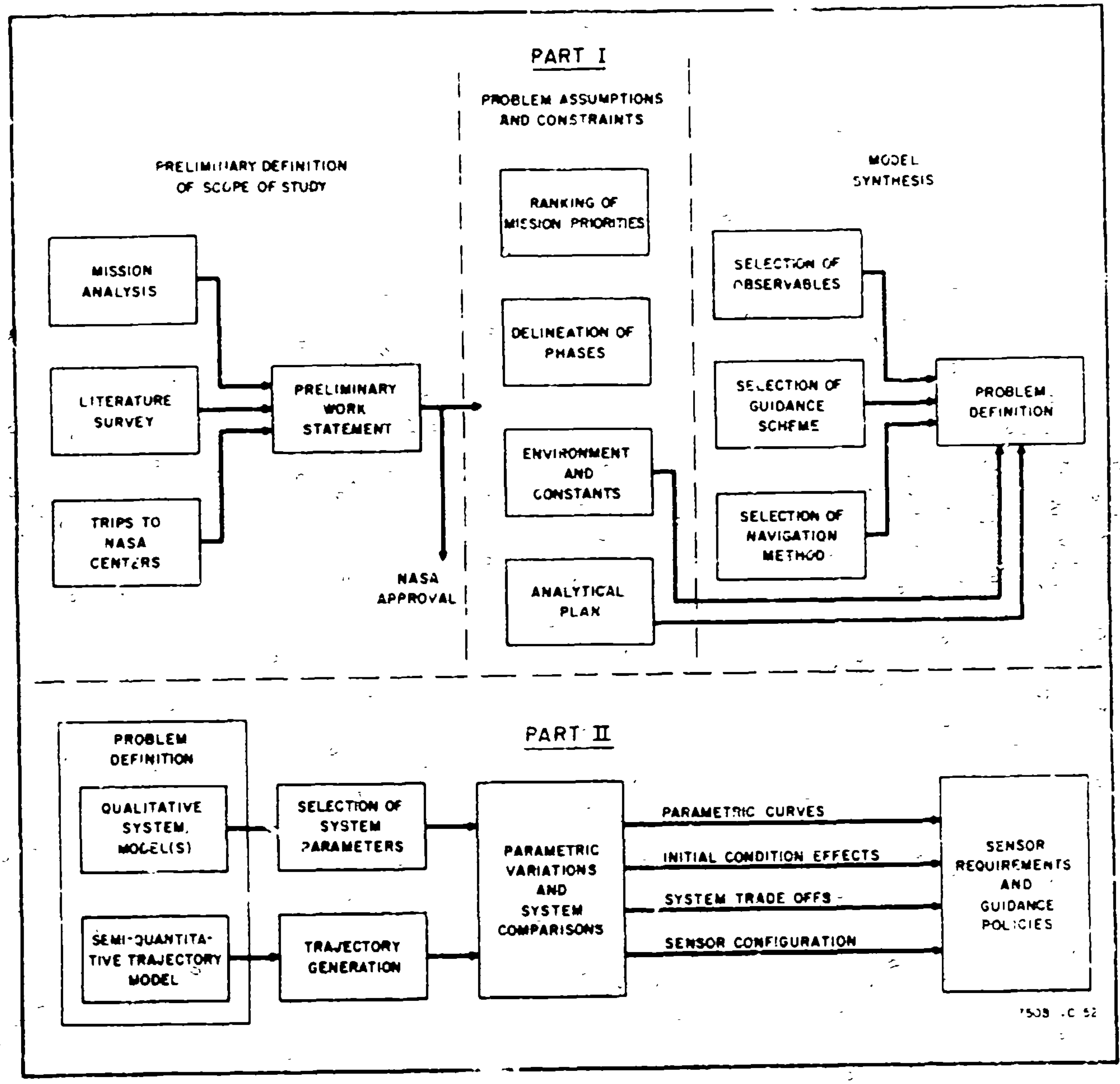


Figure 1-2. Block Diagram of Study Program

Certain assumptions were made, such as limiting launch vehicles to Saturn V (Ref. 1-2) capability, and imposing some trajectory energy constraints, in order to ensure that a realistic analysis was being conducted. In addition, environmental conditions which could affect navigation and guidance system performance are listed although these factors would have greater effect on mechanization than on analysis.

The most important outputs of the Problem Definition are the models for analysis developed for each of the five mission phases studied. These models for analysis consist of system models and trajectory models. Development of the system models entails the selection of observables for navigation and selection of the navigation and guidance procedures. Development of trajectory models is accomplished by specifying the critical trajectory parameter desired, such as lunar close approach, etc.

1.2.2 Analysis

In Part II, Analytical Solution, the semi-quantitative system and trajectory models developed in the Problem Definition are first defined explicitly, then utilized in computer programs to develop parametric tradeoffs. Figure 1-2 indicates a general outline of the plan for the analytical work. The Analytical Solution is contained in Volume III.

2. MISSIONS

In order to determine sensor requirements quantitatively, the space missions for which these sensors are applicable must be defined. The proper selection of these missions will ensure that the resulting sensor specifications are sufficient not only for the missions selected, but also for most other space missions in the time period of interest (1968-75).

2.1 GENERAL CATEGORIZATION OF MISSIONS

Although the total number of possible space missions is limitless, there are certain general characteristics of all space missions which are useful in categorizing the missions. This categorization limits the total number of missions to be considered and also illustrates the commonality of certain phases of many space missions.

Table 2-1 lists possible future manned space missions according to destination, earliest feasible launch date, mission duration, and objectives. Table 2-2 is a like categorization of unmanned missions (Ref. 2-1).

In addition to the missions listed in tables 2-1 and 2-2, one possible mission, which might be manned or unmanned, is the rescue of a space vehicle or its contents. This mission will not be considered here.

Inspection of tables 2-1 and 2-2 reveals that future space missions can be broken down into three general areas according to distance: near-earth, lunar, and interplanetary. Although the possible scientific and training experiments which may be performed on these missions are varied, they will have little effect on guidance requirements. Therefore, general mission selection can be done on the basis of the broad categorization shown in tables 2-1 and 2-2 without regard to the exact details of the various missions.

The charter of the study effort was to investigate sensor requirements for guidance of advanced space missions. Therefore, near-earth space missions other than the manned space station are not considered in the study, since they are outside the study field-of-interest and guidance requirements are expected to be simpler than for more complex missions such as lunar landing. However, the manned space station is of special interest due to the rendezvous requirements and also since it is a mission which will very likely be flown within the decade.

Table 2-1

MANNED SPACE MISSIONS

Destination	Earliest Feasible Launch Date	Duration	Purpose
Near Earth (earth orbit)	Present	Less than 2 weeks	Training Astronomical observations Equipment and personnel testing
	Late 60's	Long stay time (months - years)	Space station: Astronomical observation Meteorological study Personnel and equipment testing Communications Biological studies
Lunar (circumlunar or orbital)	1967	approximately 1 week	Surface observation Training, testing of techniques and equipment
Lunar (landing)	1968	1 week	Technique development and training. Geological samples Human observation and surveying
Lunar (lunar stay)	1970 -	several months	ALSS Lunar exploration and geodetic surveying.
Mars or Venus Flyby	1975	1 to 3 years	Technique development, close human observation of planets. Testing.
Planetary (Mars or Venus orbiter)	1980	1 to 3 years	Same as above, but over extended time.

TABLE 2-2

UNMANNED SPACE MISSIONS

Destination	Earliest Feasible Launch Date	Duration	Purpose
Near-Earth (ballistic)	Present	Minutes	Research Materials study Vehicle tests
Near Earth (orbital)	Present	Depends on satellite lifetime	Meteorological research Communications. Radiation belt study Field study Relay station
Cislunar Space	Present	Hours - months or years	Environmental observations
Lunar (hard-lander)	Present	3 days	Lunar surface photography
Lunar (soft lander)	1965	Greater than 3 days	Logistic vehicle Close surface photography Environment testing Surface hardness evaluating
Lunar (orbital)	1966	Greater than 3 days	Lunar surface photography Hidden side photography
Planet flyby orbit landing	Present '66 - '67 '68 - '69	Several months - year	Interplanetary environment study Planetary study of surface atmosphere biology fields

Required energy is also an important limitation on the types of mission which can be realized. Thus, it is assumed that manned planetary landings are beyond the scope of this study, since the energy requirements for such a mission are beyond the capabilities of launch vehicles planned for the next decade.

As a result of the factors mentioned above, primary emphasis of the study is focused upon lunar missions, the manned space station, and interplanetary missions, not including landing. In the following subsections the mission phases, mission priorities, and mission profiles utilized in this study are discussed.

2.2 MISSION PHASES

Although there are a great many possible space missions, most of them consist of several different phases, which are characterized primarily by distance from some gravitational body, atmospheric or non-atmospheric conditions, and powered or unpowered flight. The differences between phases of a mission are normally more pronounced then the differences between similar phases of different missions. For instance, the velocities, trajectory shape, propulsion methods, etc on the midcourse and landing phases of a lunar mission are quite dissimilar, while the differences between the midcourse phases of interplanetary and lunar missions are much less evident.

In general, the various phases and operations of possible space missions given in chronological order are as follows:

a. Prelaunch
b. Launch to earth orbit
c. Earth parking orbit
d. Earth orbit rendezvous
e. Launch into escape or near-escape trajectory
f. Ballistic flight to region of moon or planet
g. Planetary (or lunar) approach
h. Thrust into planetary (or lunar) orbit
i. Planetary (or lunar) orbit
j. Ballistic descent from orbit
k. Powered descent to hover
l. Descent from hover to surface
m. Ascent from surface
n. Rendezvous (to within 500 feet)
o. Rendezvous terminal docking
p. Return injection into earth-bound trajectory
q. Ballistic return to earth
r. Reentry into earth's atmosphere
s. Slow descent to earth's surface

The above list was considerably shortened and consolidated in order to map out reasonable areas of analysis. Since there has been considerable work done on requirements for items a, b, c, r, and s, these items were deleted from the list.[1] Thus, for this study, the phases of interest are those from thrust into an earth-escape trajectory until return to the earth's atmosphere. In addition, many of the phases in the above list are short enough or sufficiently similar to other phases so that they may be analyzed together. Thus, the list of phases as revised for this study is as follows:

I. Midcourse Phase	e. Launch into earth escape on near escape f. Ballistic flight to moon or planet g. Planetary (or lunar) approach q. Ballistic return flight to earth trajectory
II. Orbital Phase	h. Thrust into planetary (or lunar) orbit i. Planetary (or lunar) orbit j. Ballistic descent from orbit
III. Landing Phase	k. Powered descent to hover l. Descent from hover to surface
IV. Ascent Phase	m. Ascent from surface
V. Rendezvous Phase	n. Rendezvous (to within 500 feet) o. Rendezvous terminal docking

The breakdown of space missions into the five basic phases listed above enabled the analysis of one phase to be made independently of the analysis of each other phase. At the same time, complete mission analysis is achievable simply by matching the rms output errors at termination of one phase to the rms initial errors for the subsequent phase.

As for the missions under consideration, the phases listed are similar regardless of whether lunar or interplanetary trips are being considered. The primary differences are the greater distances involved in all phases of a planetary mission and the planet atmosphere (on Venus or Mars).

2.3 MISSION PRIORITY RATINGS

The following mission priorities were assumed for this study:

1. Manned Lunar Mission
2. Earth Orbital Rendezvous
3. Unmanned Lunar Mission
4. Manned Interplanetary Mission

[1] Conference between Westinghouse and NASA, Langley at Langley Field, Va. in 1962.

This ranking was made with the guidance of NASA Headquarters in December 1962. Some of the considerations for priority-ranking the missions are as follows: (1) mission probability, (2) mission importance, (3) difficulty and importance of guidance related to present state of the art, and (4) expected date of mission.

From all standpoints, mission 1, the Manned Lunar Mission deserves first priority since the mission will surely be attempted, the mission has great importance as a national goal, and it is an advancement of manned space exploration capability. The guidance techniques (involving rendezvous and ascent from the lunar surface) are difficult, and state of the art development is not yet sufficient to offer simple solutions to these guidance problems.

Mission 2, Earth Orbital Rendezvous, is rated second because the manned space station concept has recently been the subject of renewed interest both in NASA and in the Defense Department. It is now reasonable to assume that the manned space station will be a large post-Apollo effort. However, the Earth Orbital Rendezvous mission is rated lower than the manned lunar flight due to its less immediate timing and the fact that the mission requires only one special guidance function (i. e., rendezvous) with a near-earth space station.

Mission 3, the Unmanned Lunar Mission, rates lower than the first two in the areas of mission importance and guidance difficulty. Since the vehicle is unmanned, guidance requirements (in terms of accuracy, not mechanization) might be less severe since the possible loss of human life is not a factor. Also, it is expected that unmanned flights will utilize beacons on the lunar surface - which should considerably ease guidance problems.

Mission 4, the Manned Interplanetary Mission, rates last on all counts except difficulty of guidance. At present, it would appear that the mission will be flown only if manned lunar flights are successful and political and economic conditions in this country in the 1970's are favorable for such an ambitious space venture.

2.4 EFFECT OF PRIORITY RATINGS ON ANALYSIS

The priority rating of missions developed in the previous subsection has resulted in a modification of the phase-by-phase outline shown in subsection 2.2. In order to weigh the analytical results according to the priority ratings, the following procedure was adopted in analyzing each of the phases listed in subsection 2.2. First, a Manned Lunar Mission has been assumed for the analytical work on the phases listed. Second, since the Manned Lunar Mission and the Earth Orbital Rendezvous are dissimilar except possibly for rendezvous, the work on Earth Orbital Rendezvous is presented in a separate volume (Volume IV) of this report.

As for the Unmanned Lunar Mission, since there would be no difference between requirements for manned and unmanned guidance systems, the results obtained on the analysis of the manned mission are directly applicable to unmanned missions. The greatest differences between the missions will be in the implementation of the guidance system rather than the requirements, since the presence of a human operator on board gives the manned system a significant advantage especially because of his optical recognition ability.

The trajectories used for manned missions may differ from the unmanned flights, due to the probable elimination of lunar rendezvous on the unmanned missions. However, this will generally simplify guidance requirements. Therefore, it can be assumed that the requirements for the Manned Lunar Mission will include those for the unmanned mission.

The lowest priority mission, the Manned Interplanetary Mission, is accorded least attention. No analysis of requirements specifically for the interplanetary mission were developed. However, many of the general principles of space navigation which have been developed in study of the Manned Lunar Mission are also applicable to interplanetary flights.

2.5 MISSION PROFILE

For this study, first priority was assigned to the Manned Lunar Mission. In order to make the problem definition more concrete, a specific mission must be selected for analysis. It was decided to use a mission profile similar to that of the Apollo mission. The reasons for this choice are as follows:

a. The Apollo mission profile is relatively complex from a guidance standpoint due to the lunar rendezvous technique employed. Thus, analysis of this mission is desirable since it represents a difficult case and includes phases which are representative of almost every conceivable guidance operation.

b. Once a successful Apollo mission has been completed, succeeding manned lunar missions will probably follow mission profiles similar to the first flight, rather than devise some radically different trajectory configuration.

The mission profile which is assumed as a base line in this study is illustrated in figure 2-1 and described in the following paragraphs.

2.5.1 Midcourse Phase

The spacecraft is injected into a translunar trajectory from an earth orbit of 185-km altitude.[2] Injection occurs at a longitude of about 175 degrees from the earth-moon line at time of arrival. The midcourse trajectory is approximately coplanar with the plane of the moon's orbit about the earth. The space vehicle requires 72.2 hours to arrive at a lunar close approach (periselenum). At nominal time of periselenum, the moon is nearly at its maximum declination.

2.5.2 Lunar Orbit Phase

This phase is assumed to begin at nominal time of midcourse periselenum. At this time, a retrothrust of about 800 m/sec is initiated to place the vehicle in a 200-km orbit about the moon. The vehicle then travels ballistically for about 1 3/8 orbits while navigation data is acquired. When the lunar orbit estimation has been sufficiently refined, a lunar landing module is detached from the mother vehicle and a retrothrust is fired in order to place the lander on an elliptical path with a periselenum of 20-km about 90 degrees from the point of retrothrust. This is the so-called synchronous orbit method which allows automatic rendezvous of the two vehicles if descent to the lunar surface is not initiated.

2.5.3 Lunar Landing

When the landing vehicle arrives at a 20-km altitude, at a distance of 310 km from its landing site, the main landing engines are ignited and powered Descent Phase begins. This phase employs a near-minimum fuel-thrusting program to arrive at a hover condition (zero velocity) some 500 meters above the lunar surface. From this point, the human operator controls the descent of the vehicle to the lunar surface.

2.5.4 Lunar Ascent

After surface operations have been completed, the lunar lander is launched from the lunar surface into a 30-km parking orbit in order to arrive at the correct phasing for the rendezvous with the mother vehicle. The vehicle is powered all the way, with a pitch-program such that the burnout occurs when the vehicle has achieved a horizontal velocity equal to orbital velocity at 30 km.

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[2] Specific numerical values are used throughout this description of the nominal mission profile merely to indicate the magnitudes involved. However, the analyses (Vol. III) were not restricted to use of these exact values.

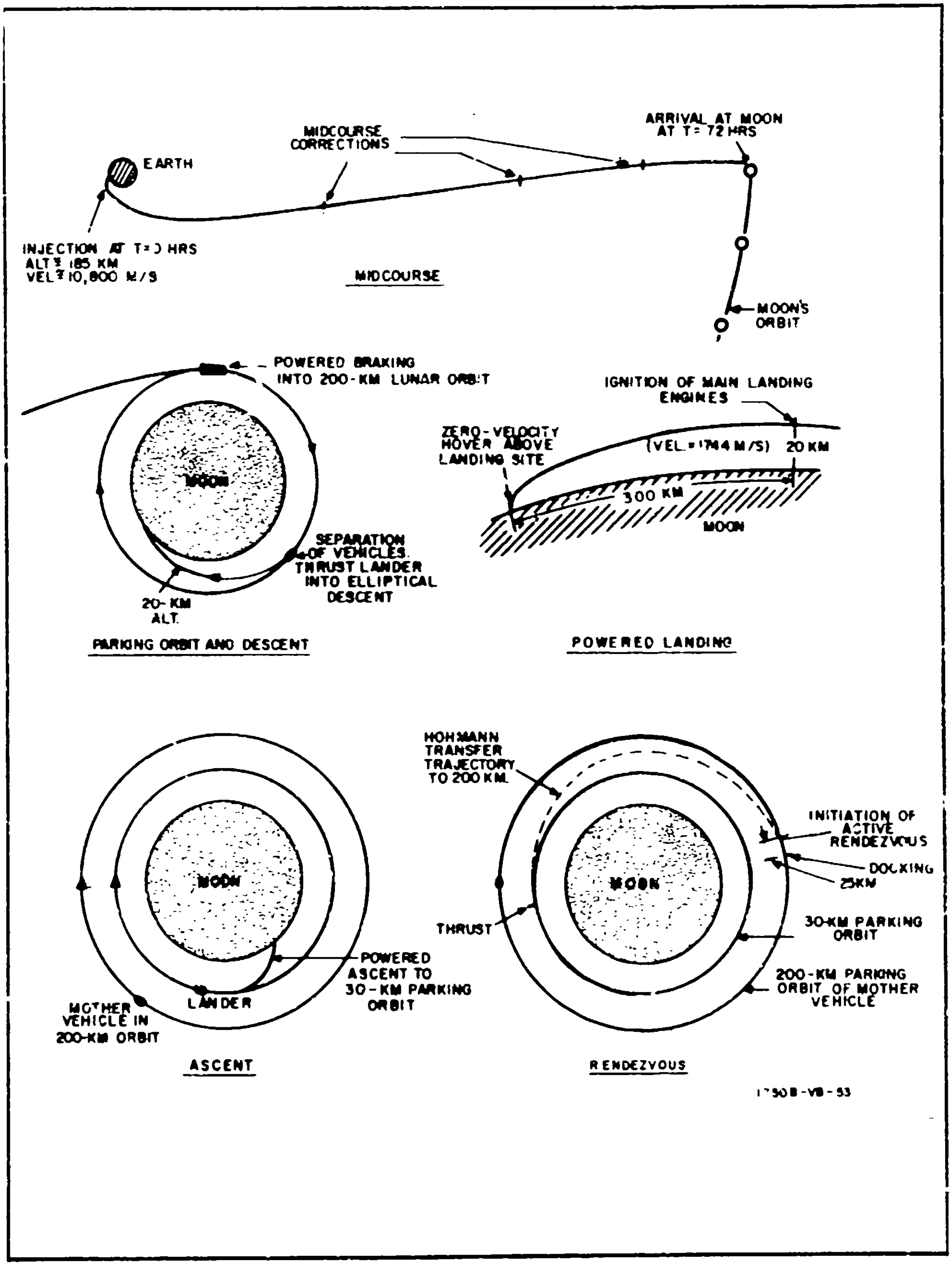

Figure 2-1. Mission Profile

2.5.5 Lunar Rendezvous

The lunar lander has been injected into a 30-km orbit compared to a 200-km orbit and thus "catches up" (in lunar central angle) to the mother vehicle. When their relative positions are correct, the lander (now the chaser) fires rockets to send itself into a transfer trajectory in order to achieve a rendezvous at 200 km with the mother (target) vehicle. This transfer is ballistic until the chaser is within 25 km of the target. Active rendezvous is then initiated and continues until the vehicles are mated. After rendezvous, the spacecraft would inject into a moon-earth transfer trajectory, but this return trip was not analyzed in the study.

Note that the only difference between the mission profile described above and the nominal Apollo mission profile is that on the first Apollo flight a direct ascent to rendezvous will be employed rather than the parking orbit mode described above. However, the direct ascent to rendezvous is also analyzed in this report.

Also it should be mentioned that, although the Apollo mission profile was used as a base for analysis, the results are not restricted to this particular mission.

3. PROBLEM STATEMENT AND STUDY OBJECTIVES

In this section a generalized statement of the fundamental guidance problem is given and the assumptions and restrictions of the study program are developed and discussed.

3.1 FUNDAMENTAL GUIDANCE PROBLEM

The primary function of the guidance system for a space vehicle is to cause the space vehicle to arrive at some prescribed terminal conditions at the end of its flight. These desired terminal conditions might include position, velocity, attitude, or attitude rate at some particular end time (fixed time of arrival). Alternately, time of arrival may not be constrained exactly, so long as the proper dynamic conditions are met by the vehicle within some reasonable time.

Other desirable conditions which should be met by the guidance system are: minimization of fuel, adherence to a preplanned "nominal" trajectory, and avoidance of conditions so drastic (e.g., high g-forces) as to damage the spacecraft or its contents.

Figure 3-1 is a functional block diagram of a space guidance system. The diagram is quite general, yet complete; i.e., any guidance system can be defined in terms of the diagram shown, and conversely, specification of all the blocks in the diagram completely defines a system.

As shown in figure 3-1, the general guidance problem can conveniently be subdivided into three primary functions: Navigation (consists of sensing observables and estimating vehicle state), Guidance Logic (consists of generating control commands to meet the desired end conditions) and Control (consists of implementation of control commands by rocket motors). Some guidance schemes may derive control signals directly from the observations without determining the "estimated state" shown in figure 3-1 as an intermediate output. This refinement is not considered important and, in general the guidance systems investigated in this study will take the form shown.

The guidance systems analyzed in this study may be conveniently classified as open-loop systems or closed-loop systems. During the Midcourse and Orbital Phases, for instance, velocity corrections will be brief and most of the time the vehicle will be in free-flight. Therefore, between velocity corrections the guidance system used in these phases is open-loop; i.e., in figure 3-1, only the "estimated state" is generated and the feedback loop (guidance logic and control) is not activated. However, during powered flight phases, such as Lunar Landing or Ascent, the steering commands continuously affect the observations so that a closed-loop system is in effect.

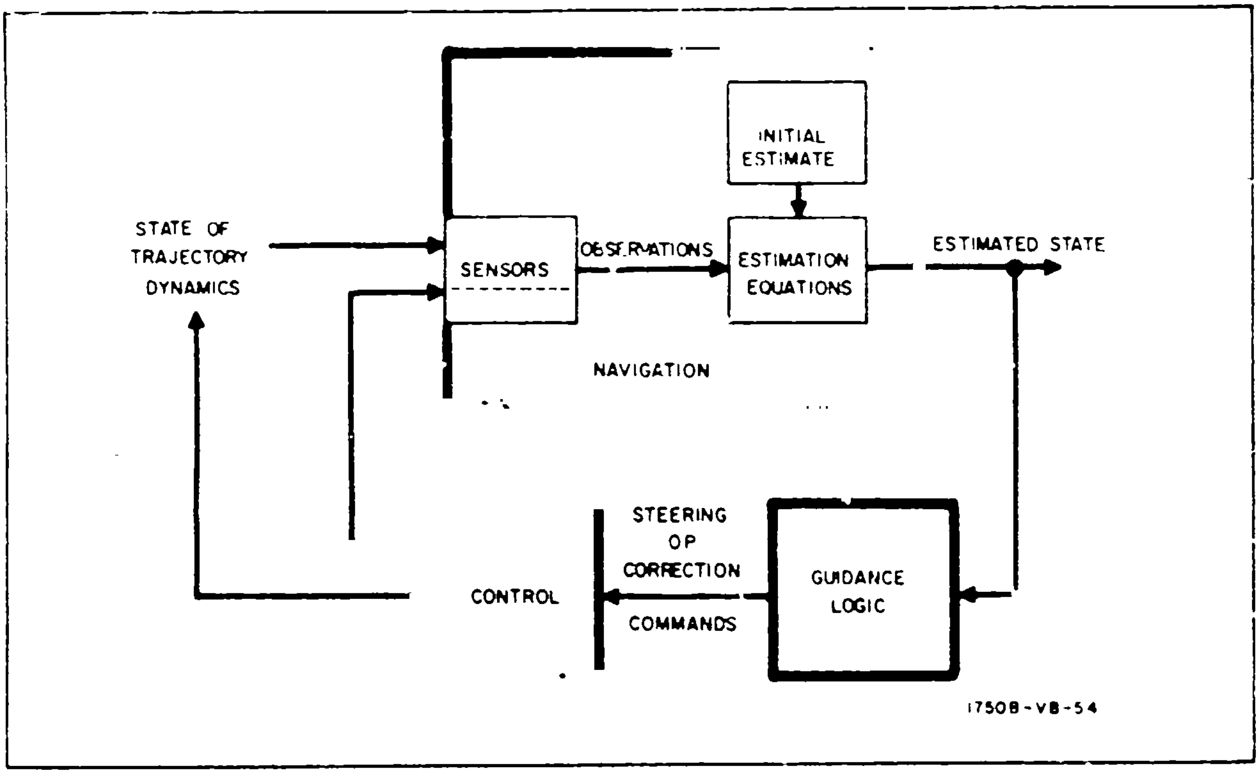

Figure 3-1. Functional Block Diagram of Generalized Space Guidance System

3.2 EMPHASIS ON NAVIGATION

In the study of the phases employing closed-loop guidance systems, primary emphasis is on the navigation aspects of the overall guidance problem rather than the guidance logic or control (see figure 3-1). This choice was made because of the desire to emphasize sensor requirements, rather than to detail control mechanization requirements. In the Lunar Landing guidance analysis, typical control systems and errors are assumed but there is no attempt to optimize these systems. The intent is to choose control systems in such a way that the resulting analytical models are reasonable.

Since the primary emphasis is on navigation rather than control, and since determination of sensor requirements are of paramount importance, attitude control requirements are not analyzed in great detail except as they are implicit in the derived results. For instance, the determination of allowable pointing error for vehicle thrusting during the Midcourse Phase implies attitude control accuracies which are within the allowable pointing error.

3.3 ONBOARD METHODS

Major emphasis in this study is on onboard guidance systems. Although there are some phases of the Manned Lunar Mission which might well be controlled from earth, such as the Midcourse and Orbital Phases, the study emphasizes analysis of onboard systems for several reasons:

a. The use of an onboard guidance system allows the use of earth tracking as a backup (and vice versa).

b. Earth-tracking accuracies have been rather thoroughly investigated by Jet Propulsion Labs, and equipment for accomplishing this tracking is already in existence (Ref. 3-1, 3-2, 3-3). Thus, further analysis in this area would tend to result in duplication.

c. Since one of the primary motivations for this study is determination of future sensor requirements, further study of ground-tracking methods is unnecessary, since these methods do not entail the development of new sensors.

d. Terminal guidance and navigation of lunar missions should be with respect to the moon, not the earth, in order to remove effects of uncertainties in location of the moon. Thus, local guidance is desirable.

Ground-tracking methods are discussed in this study only in the Midcourse Phase, and even then only as a base for comparison with onboard methods.

3.4 OBJECTIVE METHOD OF ANALYSIS

In summarizing the overall problem statement and assumptions, it is again emphasized that in this study the approach taken was as objective as possible. In other words, every attempt was made to avoid constraining the analysis by assuming specific hardware at the outset, especially in the case of navigation sensors. The analysis was always aimed at producing functional requirements for sensor equipment rather than by assuming specific equipment and then determining how well it solves a particular problem. In this way, it is felt that the results shown are generally applicable to the problem of guidance of a manned lunar mission.

The following sections consist of a more detailed discussion of the Problem Definition for each of the five subphases of the lunar mission: Midcourse, Lunar Orbital, Lunar Landing, Lunar Ascent, and Lunar Rendezvous.

4. MIDCOURSE GUIDANCE PROBLEM DEFINITION

A general statement of the guidance problem from the Problem Definition study for The Manned Lunar Mission is as follows:

"Determine the guidance system requirements and techniques necessary to achieve the guidance of a manned space vehicle to a preselected point above the lunar surface with a velocity such that appropriate landing techniques may be used. The following constraints are imposed on this guidance system: Near-minimum fuel expenditure, reliability consistent with manned operation, and compatibility with predicted post-Apollo launch and spacecraft equipment."

Although the above statement seems rather broad, note that it entails guidance to a specific location, not just the achievement of some safe lunar altitude. Note also that the requirement for compatibility with predicted post-Apollo equipment restricts the guidance situations to those arising from a Saturn V launch since this is the only presently planned vehicle capable of delivering manned vehicles to the moon in the post-Apollo period.

The following subsections on the Midcourse Phase are devoted to deriving an explicit definition of the problem from the general statement given above.

This problem definition consists of a group of trajectories for which guidance is to be accomplished (subsection 4. 2) and system models (subsection 4. 4) which are to be analyzed for performance requirements. In addition, some of the data such as astronomical quantities which are essential to the study are listed in Appendix A of this volume.

4.1 MIDCOURSE TRAJECTORY CHARACTERISTICS

To define the midcourse guidance problem quantitatively, trajectory models must be generated which are representative of the types of trajectories which might be flown on actual missions and illustrate the effects of various trajectory parameters (trip-time, etc.) on guidance system requirements.

A typical translunar trajectory for achieving retrograde lunar motion is illustrated in figure 4-1a. Part (a) of the figure shows the flight path in earth-centered coordinates. Part (b) shows the so-called approach hyperbola which results from plotting the flightpath in lunar coordinates.

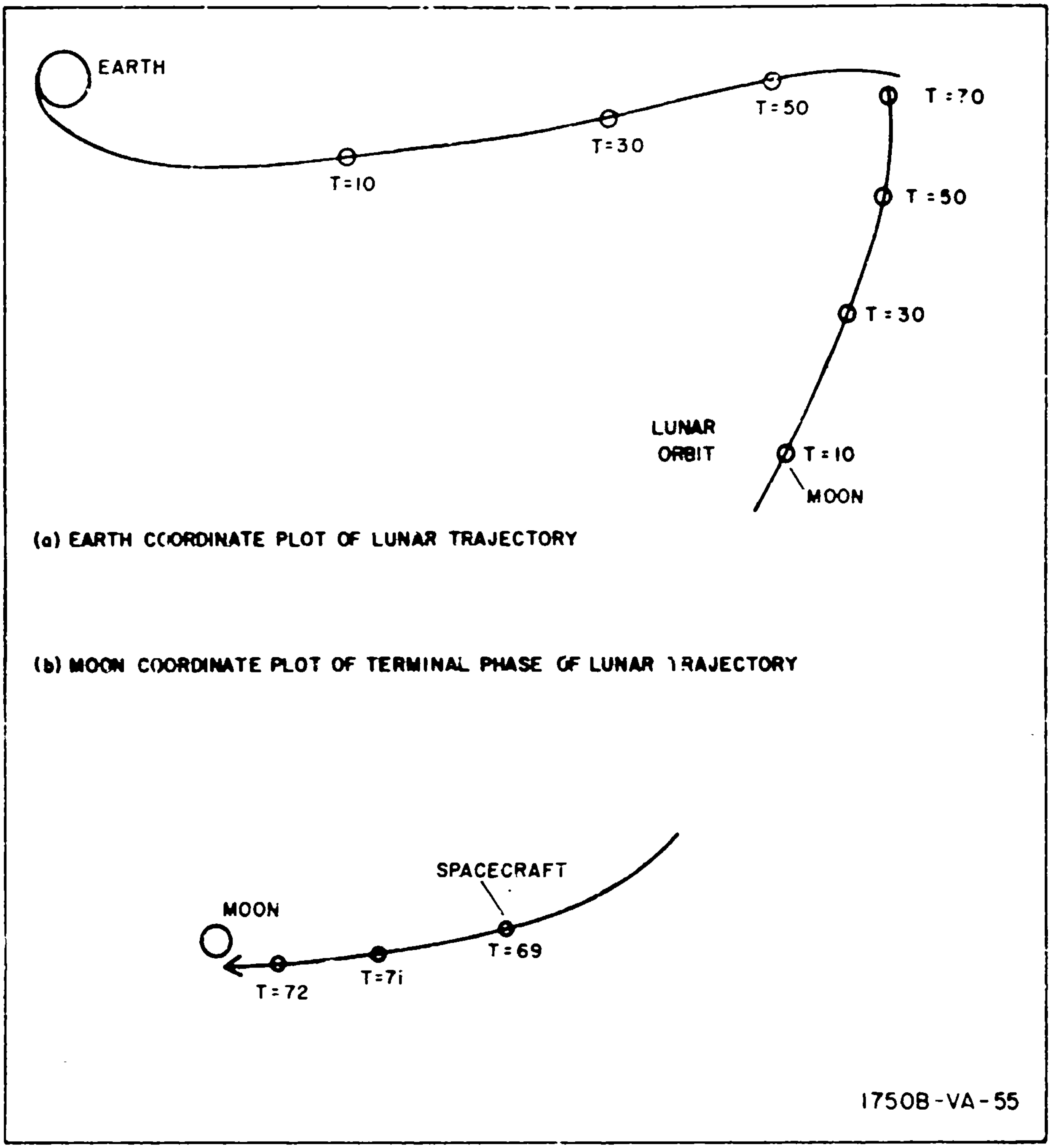

Figure 4-1. Translunar Trajectory

4.1.1 Flight Time/Energy Relationships in Midcourse

One of the factors which will affect guidance system requirements is the flight time of the mission. This is because the flight time will determine the velocity profile throughout the flight, which in turn will affect the propagation of errors. Also, the flight time determines the amount of movement of celestial bodies during the flight.

The primary considerations affecting the choice of flight time, especially for a manned mission, are the payload-energy requirements, which make a slower flight more desirable, and the life-support requirements, which favor use of a shorter flight. Although the purpose of this study is to determine sensor requirements rather than to optimize flight times, a good idea of the flight time/energy tradeoff is given in figure 4-2 (Ref. 4-1). It can be seen that for flight times above about 60 hours, the trip time increases very rapidly with decreasing burnout velocity. It is apparent, then that there is a lower limit on the burnout velocity required to keep the trip-time reasonable For the initial Apollo flight, the flight time has been selected as approximately 72 hours.

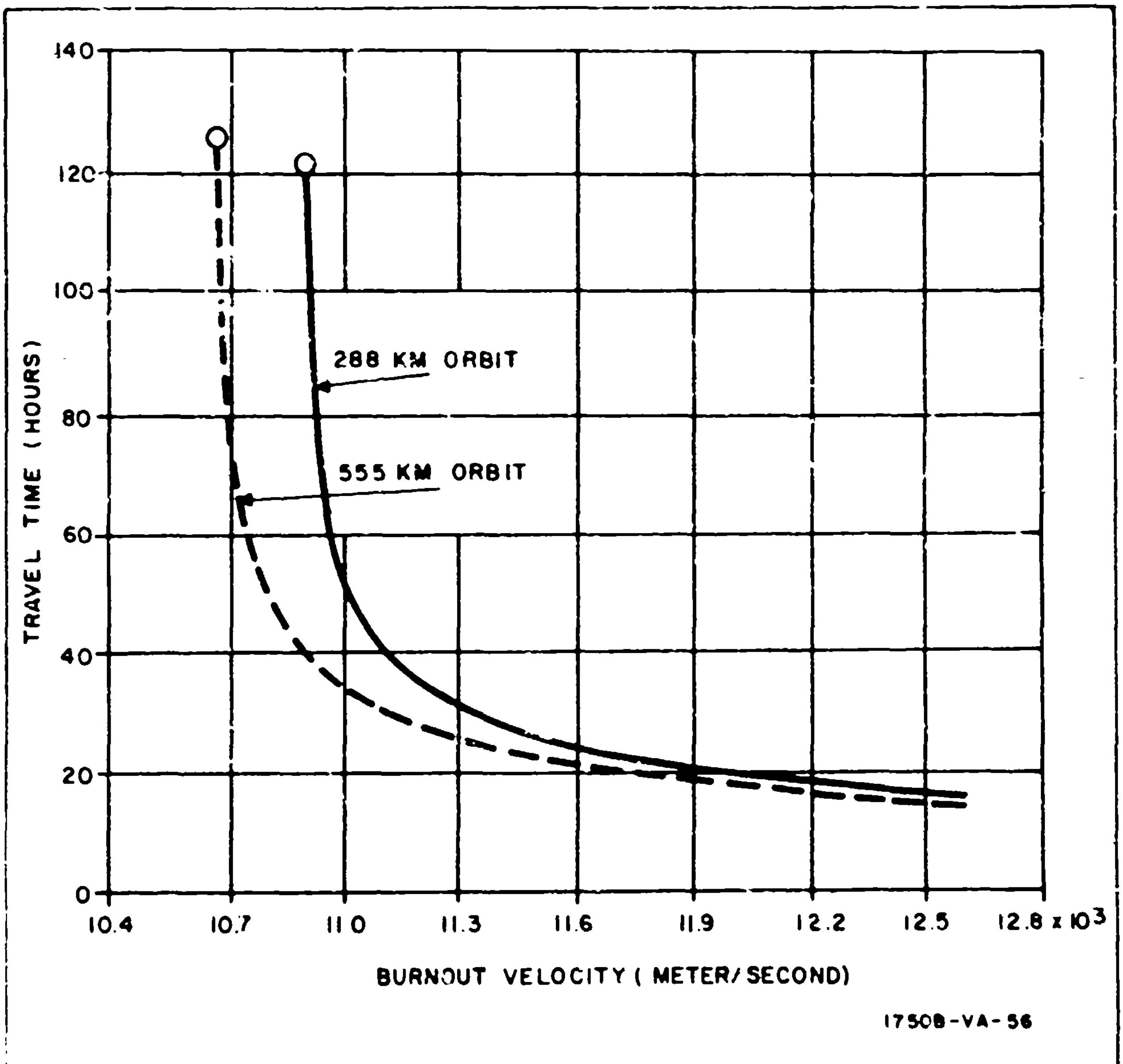

Figure 4-2. Trip Time as a Function of Burnout Velocity

4.1.2 Injection Altitude

Most presently available data on the Saturn V launch vehicle (Ref. 4-2) is based on a 185-km earth parking orbit for lunar flights. Therefore, it is assumed in this study that midcourse injection will occur from this altitude.

4.1.3 Trajectory Plane Orientation

The nominal inclination of the midcourse trajectory plane depends to some extent on the desired inclination of the orbit achieved at the moon. In the midcourse study it is assumed that an equatorial orbit around the moon is desired.*

There are several reasons why it is desirable to achieve a lunar orbit which is in the plane of the lunar equator. For one thing, it makes the use of lunar rendezvous techniques simpler, since even if a vehicle separates and descends to some landing point on the equator, both vehicles will remain in a nearly equatorial plane and rendezvous can be accomplished with no expensive plane-changes. This is not true for orbits which are inclined an appreciable amount to the lunar equator. Another reason for choosing an equatorial lunar orbit is that the region near the equator is better mapped than the higher latitude regions and guidance and landing operations in the equatorial region will be less uncertain.

The simplest way to achieve an orbit around the moon's equator without requiring any plane-changing thrusts is to send the space vehicle on a trajectory which is coplanar with the plane of the moon's orbit about the earth. If this is done, and the space vehicle arrives in the vicinity of the lunar equator, then the result is a flight path which is concentric and coplanar with the lunar equator such that a lunar equatorial orbit may be achieved by retrothrusting at any time in the trajectory without requiring a plane change. This is illustrated in figure 4-3.

Since the minimum inclination orbit achievable is equal to the latitude of the launch site, then an in-plane launch from Cape Kennedy can be achieved only when the plane of the moon's orbit is inclined to the earth's equator by more than 28.5 degrees. During the years 1968-1975, the inclination of the lunar plane varies from 25 to 30 degrees (Ref. 4-3) so that a nearly in-plane launch can always be achieved during this period.

Use of a parking (or coasting) orbit before launch considerably reduces the launch window restrictions on day and time as shown in Ref. 4-4. However, since injection into the lunar transfer trajectory must occur some 175 degrees in longitude from the earth - moon line at time-of-arrival, launch to a southerly declination moon is desirable if injection is to occur over the Atlantic Ocean. At any rate, in this study, the only concern is with the

* In the analysis of the Orbital and Landing Phases, the orbital inclination does not affect the analysis and the assumption of an equatorial orbit around the moon is not made.

inclination of the midcourse trajectory plane, not with how the particular trajectory is achieved, except for the case of ground tracking.

For attaining lunar orbits inclined to the lunar equator, it is clear that this can also be achieved with an in-plane launch by aiming at some place other than the region of the lunar equator, as shown by the dashed trajectory in figure 4-3.

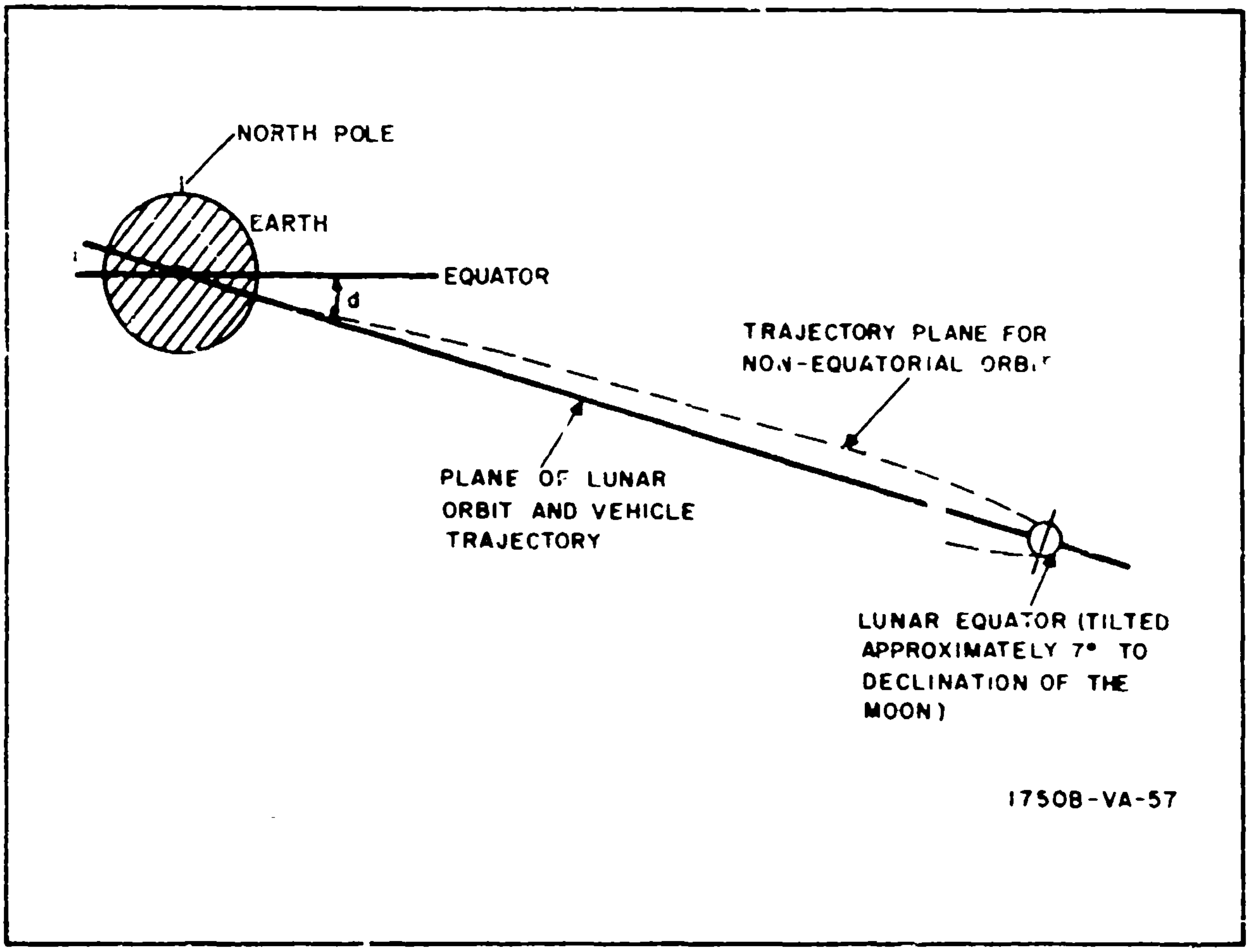

Figure 4-3 In-Plane Trajectory to Orbit Around Lunar Equator

4.1.4 Target Altitude at Moon

To achieve a lunar altitude of x km in the most economical manner possible, the midcourse trajectory should be designed so that the nominal altitude of close approach is x km. Therefore, the choice of target altitude at the moon involves a choice of desired lunar orbital altitude. Although requirements may vary from mission to mission, it would appear desirable to keep this altitude as low as possible consistent with mission safety in order to allow optical surveillance from the spacecraft. Nominal lunar orbital altitudes of 100 km up to thousands of kilometers have been considered in other studies but no definite reasons are given for higher altitudes other than large guidance errors. Therefore, in this study only low-altitude orbits are considered.

As far as total velocity requirements are concerned, there is no particular advantage in any given altitude, since in any case, the requirement is to bring the vehicle to zero velocity at some hover altitude above the lunar surface. The velocity change required for this task is a function only of the energy brought by the vehicle into the lunar gravitational field; i. e., a function of trip-time not orbital altitude. Another way of saying this is that the energy added to the vehicle depends upon the distance toward the potential source attained regardless of the particular path taken.

However, total correction velocity and fuel consumed are not linearly related if the correction takes an appreciable amount of time (i. e., is non-impulsive). This means that for some important practical cases (like the constant-thrust gravity-turn landing technique), the required thrust level depends on the initial altitude and a greater payload can be delivered using a relatively high-thrust engine starting at a low altitude than a low-thrust engine from a higher altitude. This is another reason for trying to make the midcourse target at as low an altitude as possible consistent with mission safety.

4.2 TRAJECTORY MODELS

The previous subsection discusses some of the trajectory characteristics which could conceivably affect guidance requirements. The considerations mentioned were used to generate requirements on trajectories which fulfill the following requirements: (1) represent typical trajectories and (2) illustrate the effects of variation of trajectory parameters on guidance system requirements. The trajectories chosen are discussed in the following paragraphs.

- Midcourse Trajectory I

As an example of a typical trajectory which might be utilized on a manned lunar mission, a trajectory having the following characteristics was generated:

Launch Altitude	Trip-Time	Periselenum Altitude	i	ψ	ϕ
185 km	72.2 hours	200 km	0°	-138.475°	27.55°

Periselenum = Point of close approach at moon

i = angle between trajectory plane and lunar orbit plane

ψ = angle between earth-moon line at launch and earth-moon line at zero lunar declination (ascending).

ϕ = angle between lunar orbit plane and equatorial plane.

The numbers used here are considered to be typical values such as those which might be used on the Apollo mission reference. The lunar inclination to the equator, ϕ, of 27.55 degrees occurs during the years 1966 and 1972. The value $\psi = -138.475$ degrees corresponds to injection over the North Atlantic after a short coasting orbit for arrival at the moon near maximum negative declination. The approximate Apollo flight time is 72 hours. The choice of a 200-km periselenum altitude was made arbitrarily, but this parameter is varied in other trajectory models.

- Midcourse Trajectory II

To determine the effect of shorter trip times (and higher velocities) on guidance requirements, a trajectory requiring only 63.9 hours to arrive at a 133-km periselenum was generated. In other respects, this model is similar to Trajectory I; i.e., it is an in-plane flight to the moon arriving near maximum negative declination.

- Midcourse Trajectory III

To determine the effect on guidance requirements of having a trajectory out of the lunar orbit plane, a model trajectory in which the flight path makes an angle of 27.5 degrees with the plane of the moon's orbit about the earth was generated. This trajectory is similar in other respects to Trajectory I, except that the moon is near zero declination (descending) instead of maximum negative declination when the spacecraft arrives.

- Midcourse Trajectory IV

To evaluate the effect of variations in the periselenum altitude on the guidance accuracy, a trajectory was generated which is identical to Trajectory I except that a periselenum altitude of 100 km was used instead of 200 km. This might be expected to increase guidance errors, since the lower periselenum altitude will entail higher approach velocities and thus, greater errors.

4.3 POSSIBLE GUIDANCE SYSTEMS

Figure 4-4 is a functional block diagram of a space guidance system. The diagram is quite general, yet complete; i.e., any guidance system can be defined in terms of the diagram shown, and conversely, specification of all the blocks in the diagram completely defines a system. Thus, setting up a mission model requires choice of: observables, sensors, methods of estimation, guidance logic, control mechanization and control monitoring (if any). In the following paragraphs, these areas are examined and discussed in order to determine the possible system models. In addition, potential problem areas and methods of analysis are identified for the models chosen.

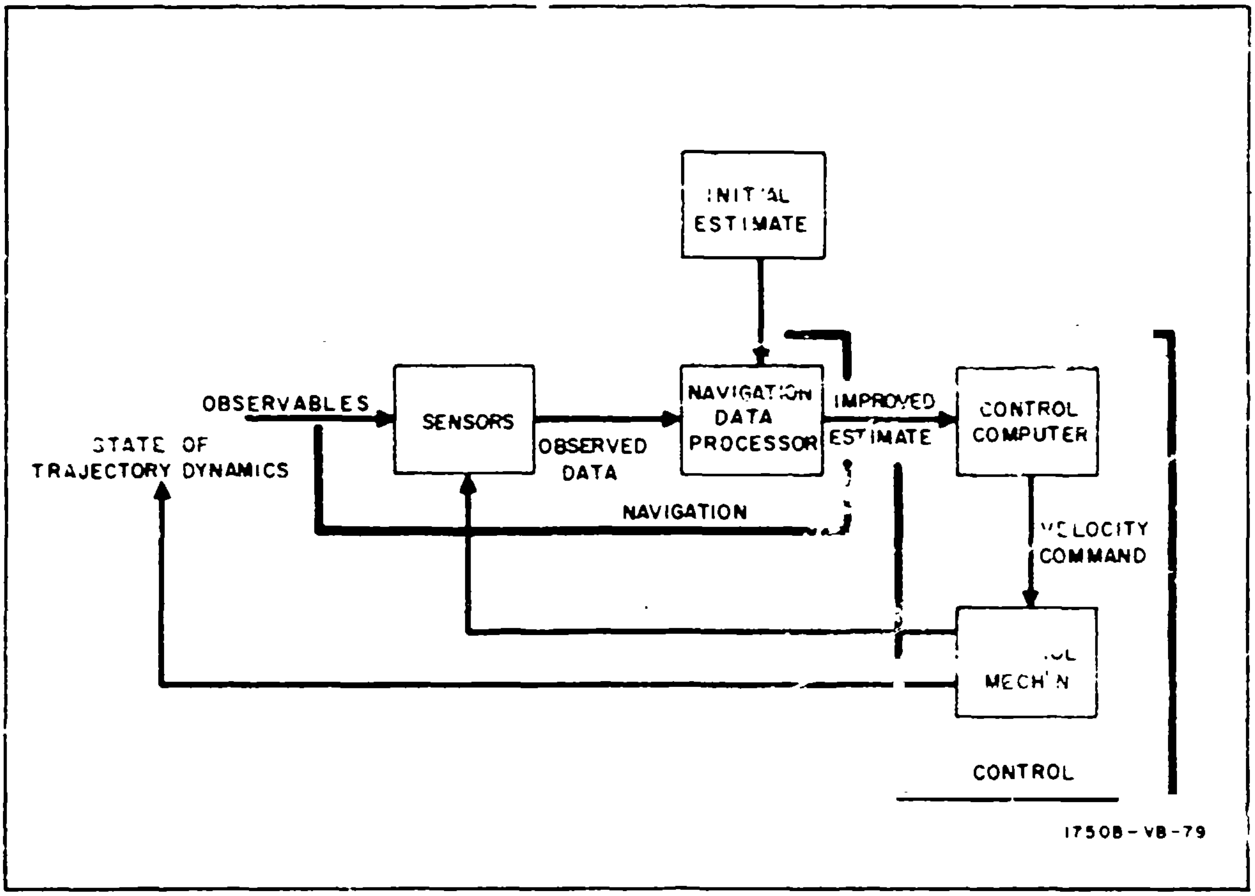

Figure 4-4. Functional Block Diagram of Generalized Space Guidance System

4.3.1 Observables

Since the navigation problem consists of determining the translational state of the vehicle; i.e., its position and velocity with respect to the planets of interest (earth and moon), the only useful observables are those physical quantities which directly relate to, or are themselves range and velocity.

Attitude measurements (such as might be obtained from star sightings only) can in no way contribute to knowledge of the vehicle's translational state, and they are essential only for control purposes. Thus, the list of possible observables useful in midcourse navigation is short:

Range (to earth or moon)
Range-rate (to earth or moon)
Velocity-rate (to earth or moon)
Angles between earth or moon and stars

The above list does not include measurements of the angles between stars as they are too far from the earth-moon system to be of any use in determining position. However, this great distance makes stars useful as coordinate references, since they will always appear in the same relative direction. Range-rate to stars (by doppler shift of starlight) is not considered, as this process is evidently not sufficiently accurate for space navigation. Finally, angle-rate is not considered either, as this quantity is too small to be measurable throughout most of the midcourse flight.

4.3.2 Methods of Observation

The known methods of observing the quantities listed in the previous paragraph are listed below in order to indicate feasibility of utilizing various observables:

Range:

Optical measurement of angle subtended by earth or moon.
Ground tracking of beacon or transponder on spacecraft.
Timing of radar echo from earth or moon.

Velocity:

Ground tracking of beacon or transponder on spacecraft.
Active doppler radar in spacecraft.

Velocity Rate:

Accelerometers (for accelerations other than gravitational).

Angles Between Earth or Moon and Stars:

Measurement of antenna azimuth and elevation angles on ground tracking system.

Optical measurement of angles between star direction and reference point (horizon or landmark) on earth or moon.

Note that this last item includes the measurement of individual space angles at various times. It is important to mention that there is no requirement for simultaneous measurement of space angles if the appropriate mathematical methods are used on the data, i.e., position "fixes" are not required (See e.g., Ref. 4-5).

In the above listing, methods of preserving an angular orientation by using inertial devices[1] are not considered separately, since these methods essentially measure the two sides of an angle by optical methods at different times and are thus no different, in principle, than the direct measurement of the space angle at once.

It is also important to note that accelerometers can measure only accelerations other than the gravitational field of interest and thus can supply no information about the vehicle's translational state in a free-flight condition. These devices can be used only to monitor velocity control actions.

4.3.3 Methods of Navigation Computation

Figure 4-4 shows that the space guidance problem is actually a feedback control loop, with observations of the vehicle's translational state being used to compute control commands which, in turn, affect the translational state. However, during the entire midcourse flight between the earth and the moon, only a few short-duration velocity corrections are required, so that most of the time the feedback loop of figure 4-1 is operating open-loop. Since this is the case, the navigation, guidance logic and control functions can be analyzed separately since they will have little affect on each other.

The navigation aspect of the problem can be simply stated as follows: Given some initial estimate of the vehicle's translational state, and some subsequent observations of the state, determine an improved estimate of the vehicle's present state and its state at some future time. It is clear that both of these estimates are required in order to generate the proper control commands to achieve the desired target conditions.

The motion of a small mass in an XYZ coordinate system is governed by equations which can be represented by the following:

---

[1] Such as using two star directions to establish an inertial reference, then measuring the azimuth and elevation of a planet with respect to this reference.

$$\ddot{X} = f_1 (X, Y, Z, t)$$

$$\ddot{Y} = f_2 (X, Y, Z, t) \qquad (4\text{-}1)$$

$$\ddot{Z} = f_3 (X, Y, Z, t)$$

Equations 4-1 are functional representations of the nonlinear differential equations of motion. When more than one attracting body exerts a significant gravitational pull on the space vehicle, such as the earth and moon, there is no explicit solution to equations 4-1 except by numerical methods. As a result, the usual practice is to linearize equations 4-1 about some reference trajectory. Expanding equation 4-1 about some reference trajectory (r) at some time, t, there is:

$$(\ddot{X} + \Delta\ddot{X}) = f_1 + \left(\frac{\partial f_1}{\partial X}\right) \Delta X + \left(\frac{\partial f_1}{\partial Y}\right) \Delta Y + \left(\frac{\partial f_1}{\partial Z}\right) \Delta Z + \text{higher order terms}$$

$$(\ddot{Y} + \Delta\ddot{Y}) = f_2 + \left(\frac{\partial f_2}{\partial X}\right) \Delta X + \left(\frac{\partial f_2}{\partial Y}\right) \Delta Y + \left(\frac{\partial f_2}{\partial Z}\right) \Delta Z + \text{higher order terms} \qquad (4\text{-}2)$$

$$(\ddot{Z} + \Delta\ddot{Z}) = f_3 + \left(\frac{\partial f_3}{\partial X}\right) \Delta X + \left(\frac{\partial f_3}{\partial Y}\right) \Delta Y + \left(\frac{\partial f_3}{\partial Z}\right) \Delta Z + \text{higher order terms}$$

where $\Delta X = (X - X_r)$, $\Delta Y = (Y - Y_r)$, $\Delta Z = (Z - Z_r)$ where X_r, Y_r, Z_r are the reference coordinates at time t and X, Y, Z, are the actual coordinates. Neglecting the higher order terms and subtracting equation 4-1 from equation 4-2, gives:

$$\Delta\ddot{X} = \left(\frac{\partial f_1}{\partial X}\right) \Delta X + \left(\frac{\partial f_1}{\partial Y}\right) \Delta Y + \left(\frac{\partial f_1}{\partial Z}\right) \Delta Z$$

$$\Delta\ddot{Y} = \left(\frac{\partial f_2}{\partial X}\right) \Delta X + \left(\frac{\partial f_2}{\partial Y}\right) \Delta Y + \left(\frac{\partial f_2}{\partial Z}\right) \Delta Z \qquad (4\text{-}3)$$

$$\Delta\ddot{Z} = \left(\frac{\partial f_3}{\partial X}\right) \Delta X + \left(\frac{\partial f_3}{\partial Y}\right) \Delta Y + \left(\frac{\partial f_3}{\partial Z}\right) \Delta Z$$

Equations 4-3 are linear approximations to equations 4-1 where the unknowns, ΔX, ΔY, ΔZ are deviations from some reference coordinates rather than the true coordinates themselves. It has been found (Ref. 4-6, among others) that as long as the true coordinates are relatively close to the reference coordinates (i.e., not an abort condition), then linear approximations such as equations 4-3 can be used effectively. The formulation of equations 4-3 was used in this study, thus solving the problem of finding useful mathematical approximations to equations 4-1.

Another mathematical problem in space navigation is the fact that the observations of the trajectory (and therefore, the estimates) will never be perfect. Since this is the case, statistical methods should be employed to utilize the data in some optimum way in order to obtain the best possible trajectory estimate.

Another reason for employing statistical methods in space guidance is that such methods are convenient for handling partial data and it may be quite difficult to obtain a complete position fix on any one observation. One method of taking a position fix is to simultaneously measure the range to a planet and the azimuth and elevation of the planet in some inertial coordinate system. Obviously, this job would be easier if these measurements could be made separately. Thus, some method of handling partial data (e.g., range or a single angle) is desirable.

These problems were solved by JPL for their determination of trajectories by use of the so-called weighted least-square technique. This process, which is described more completely in Ref. 4-7, consists of estimating the vehicle trajectory based on all the observed data (range-rate and angle at the tracking stations) and the estimate of initial conditions and measurement errors. One problem with using this method is that the large amount of data which must be handled simultaneously makes computer requirements formidable.

What was needed in this situation was a recursive method of data-processing, in which an estimate of the vehicle state could be made using only the previous estimate and the new bit of data. This problem was solved in 1961 by S. F. Schmidt and G. L. Smith of Ames Research Center (Refs. 4-6 and 4-8). These investigators applied some modern notions of linear control theory developed by R. E. Kalman (Ref. 4-9) who had developed a general minimum variance solution to the Wiener filtering problem. This minimum variance solution consisted of a set of recursive equations which were applied by Smith and Schmidt, to the trajectory estimation problem.

It turned out that the trajectory estimate provided by the minimum variance method utilized at Ames was identical to the estimate obtained by the weighted least-squares procedure under certain conditions (Ref 4-10). The difference, however, is that the weighted least-squares procedure requires the inversion of a matrix whose dimensions are as large as the total number of data points being processed, and even for large land-based computers, this operation can become intractable when correlated noise is present on the observations (Ref. 4-10). Thus, the recursive procedure is advantageous from a computational standpoint. However, an advantage of the weighted least-squares formulation is that the data may be recycled several times to eliminate the effects of blunder points; i.e., data points which are obviously the result of some gross error in the system. (There is, however, no real reason why recycling could not be done with the minimum variance method, although then the scheme would lose some of its computational simplicity.)

Another data processing method, simple least-squares, is less efficient, since this method utilizes only the actual observations from which to make an estimate. This is in contrast to the minimum variance and weighted least-squares methods, both of which utilize initial trajectory estimates and estimates of the observation errors in order to obtain the new estimate.

A more detailed mathematical discussion of data-processing for space navigation is given in Volume V, Appendix A. A mathematical analysis of some of these topics is given in Volume V, Appendix B. Section 8, and Volume III, paragraph 2.3.1.3.

4.3.4 Guidance Logic, Control Mechanization, and Control Monitoring

For the purposes of this study, it will be assumed that control of the vehicle's midcourse trajectory is supplied entirely by a high-thrust rocket motor; i.e., one capable of making nearly impulsive midcourse velocity corrections. Continuous-thrust control (ion engines, etc.) is assumed to be outside the scope of this study.

Evidently the only direct method of monitoring the control velocity change is measurement with accelerometers. Other methods of monitoring the rocket motor operation are inherently less direct since they measure the action of the motor rather than the effect of this action which is of real interest. In any case, the effect of velocity correction monitoring is small, as is shown in Volume III, paragraph 2.4.3.2.

Guidance schemes for generation of velocity correction commands can conveniently be grouped according to whether they are fixed time of arrival or variable time of arrival. In fixed time of arrival (FTOA) schemes, the three components of correction velocity are applied in such a way as to pull out the three components of estimated target position miss at the preselected time of arrival. Variable time of arrival (VTOA) schemes might take various forms, but one method is to attempt to reduce the indicated target miss in only two directions (e.g., altitude and cross-range distance at the moon) while allowing the downrange miss, which is equivalent to time of arrival, to vary. In this way, one degree of freedom is left unspecified, which may be used to minimize the required thrust magnitude or perform some other optimization.

In addition to the guidance logic required to compute the magnitude and direction of each velocity correction, the selection of correction times must be decided. Selection of correction times, together with the type of correction to be made, has been a favorite topic of mathematicians for some time. However, there is no evidence that the highly complex optimization schemes which have been generated are really required for lunar flight. For instance, in this document and Ref. 4-8, it has been found that using a simple fixed time of arrival guidance logic and trial-and-error selection of correction times, the total corrective velocity required on outbound lunar flights is only 20-30 m/sec and considerably less on the return trip. In Ref. 4-5, an attempt was made to mathematically optimize the times at which FTOA corrections should be made, but the results were not particularly good. Thus, in this study it is assumed that velocity corrections are made at fixed, preselected time during the flight.

4.4 SYSTEM MODELS

In the previous subsection, the essential elements of a space guidance system have been discussed, emphasizing the methods which would be useful during the Midcourse Phase of a Manned Lunar Mission. In this subsection, the system models which will be analyzed are discussed and the reasons for choice of these models are given.

4.4.1 Model Selection

The categorization of system models according to observables and sensor methods, which was given in paragraph 4.3.2, is repeated here for convenience:

Range:

Optical measurement of angle subtended by earth or moon
Ground tracking of beacon or transponder
Timing of radar echo from earth or moon

Velocity:

Ground tracking of beacon or transponder
Active doppler radar in spacecraft

Velocity Rate:

Accelerometers (for accelerations other than gravitational)

Angles Between Earth and Moon:

Measurement of antenna azimuth and elevation angles on ground tracking system

Optical measurement of angles between star direction and reference point (horizon or landmarks) on earth or moon

Examination of the above list shows that all the methods mentioned may be grouped under three general categories:

a. Ground tracking of signal from missile transponder or beacon
b. Onboard passive (optical) measurement of space angles
c. Onboard active radar measurements of range, and range-rate

(The above list excludes accelerometers which yield no new information during free-flight.)

Of the three possible methods, principal study effort is devoted to (b) onboard passive measurements of space angles, for several reasons. Onboard active measurement of range or range-rate does not look attractive due to the large power requirements which are involved in trying to transmit over cislunar distances which may range up to 175,000 km to the nearest planet. Power calculations done later in the study indicate that the power and dish-size requirements for accurate (±10 km) microwave ranging off the lunar surface at this distance are enormous (typically 40 kw peak power with a one-meter antenna). This does not completely rule out active radar ranging during midcourse, since at closer ranges it may be useful, such as in combination with the optical system. However, the power and antenna requirements would preclude active onboard ranging as the sole navigation input unless no other method existed.

Ground tracking techniques including tracking of spacecraft beacon and transponder signals have been demonstrated to be excellent orbital determination methods by JPL on their Ranger and Mariner programs. However as pointed out in subsection 3.2, an extensive analysis of ground-tracking from the earth would be pointless for this study, since this work has already been covered in many fine papers from JPL (e.g., Ref. 4-11, 4-7, and 4-12).

Although the feasibility of onboard optical methods has been demonstrated in Ref. 4-5 and 4-6, these analyses have been primarily concerned with the statistical data-processing aspects of the problem. In addition, many of the assumptions made in these studies have not been particularly applicable to manned lunar landing missions.

While emphasis in this study is on passive onboard methods, some work is done on ground-tracking systems for comparison, although most of this information is obtained from references. Onboard active systems are not considered, except as additions to an optical system during certain limited periods of the flight. The onboard and ground tracking systems assumed as system models are described in the following paragraphs.

4.4.2 Onboard System Model

The principal onboard guidance system to be investigated is described in table 4-1 and figure 4-5.

TABLE 4-1

ONBOARD GUIDANCE SYSTEM MODEL FOR MIDCOURSE PHASE

Observables	Single angle between star direction and landmark or horizon on earth or moon
Sensor Device	Onboard optical instrument
Navigation Method	Use of minimum-variance technique to statistically weight data Use of linearized deviation equations about reference trajectory
Control Mechanization	High-thrust rocket motor
Guidance Logic	Fixed time of arrival
Control Monitoring	Onboard accelerometers (3-axis)

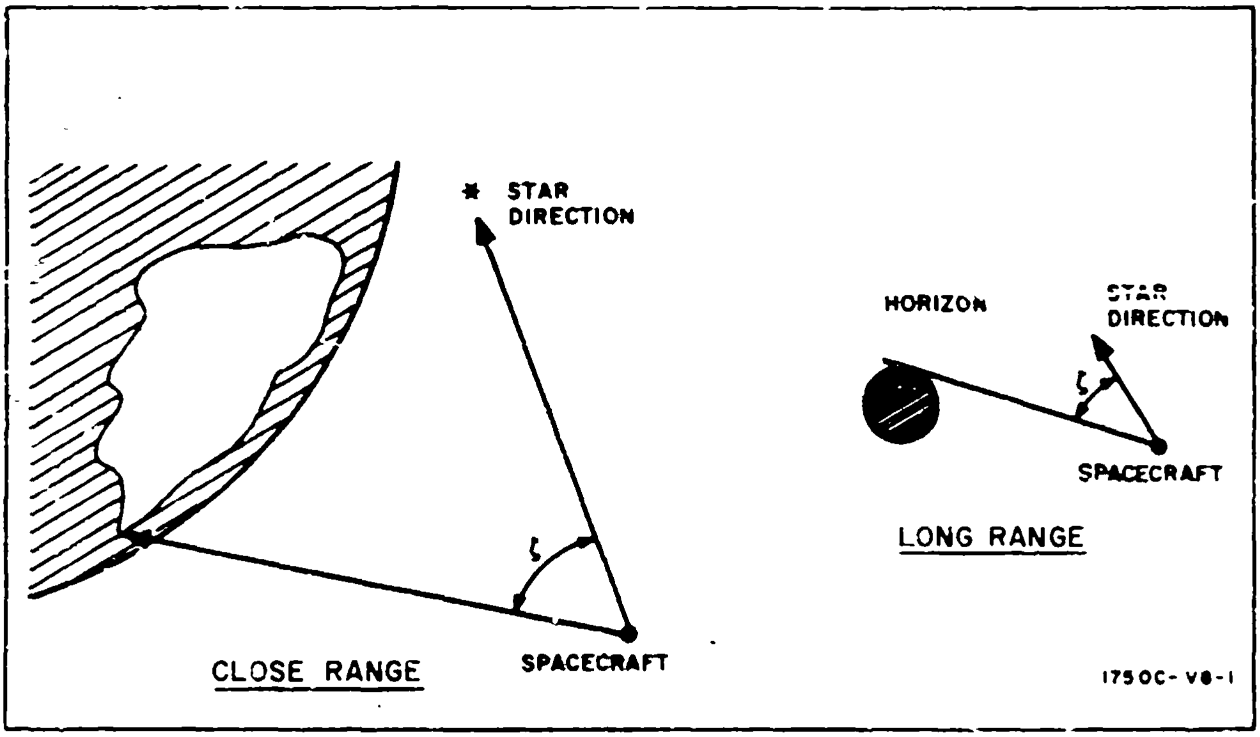

Figure 4-5. Angle Measurement for Onboard System Model

Although the primary observables chosen for analysis are the angles between some reference point and a star direction, the model formulation is extended to include a theodolite-type device, which measures the azimuth and elevation of a planet in an inertial coordinate system. This is the same as measuring two single angles simultaneously.

Note that the choice of observables depends on the distance from the planet involved. This is because at close ranges, a landmark such as a small island may be more easy to define visually than the horizon, while at long distances, the horizon may be the only readily distinguishable feature of the planet.

In Ref. 4-5 the feasibility of a single-angle device (sextant) having an accuracy of 10 arc-seconds rms is shown, while in Ref. 4-8 a three-angle device (theodolite) is assumed having an rms accuracy of 10 arc-seconds in each of three measurements, azimuth, elevation, and subtense angle. However, it is shown in Ref. 4-8 that the measurement of the subtense angle contributes little, so this measurement is not considered in this study.

The use of the minimum variance trajectory estimation method is assumed. As far as the analysis is concerned, use of either this method or weighted least squares is equivalent. Since minimum variance is more amenable to onboard computation, and also more convenient to analyze, it will be assumed that this method of trajectory estimation is used.

Fixed time of arrival (FTOA) guidance logic is assumed because of its simplicity, both in onboard implementation and for analysis.

4.4.3 Ground-Tracking Model

The ground tracking model assumed is described in table 4-2. This model was used in an analysis of ground tracking methods in Ref. 4-13, and is shown here only to describe the ground-tracking system model whose performance is compared with the onboard system model analyzed in Volume III.

TABLE 4-2

GROUND TRACKING MODEL (REFERENCE 4-13)

Stations	(3) Johannesburg, Rosman, Carnarvon (6) Johannesburg, Rosman, Carnarvon, Hawaii, Houston, Madrid
Observables	Transponder range and range-rate
Accuracies	10m - 24 meters (1 σ Range Accuracy) 0.077 - 0.237 m/s (1 σ Range Rate Accuracy)
Data Rates	(1 pt/min and 1 pt/10 min.)

4.5 ANALYTICAL APPROACH

The analytical approach for this study is directed toward determining requirements for onboard optical navigation, with ground tracking considered only for comparison. An outline of the analytical approach is detailed below:

a. Generation of Trajectories - Numerical computation of the four earth-moon trajectories listed in subsection 4.2.

b. Development of System Equations and Computer Program - Development of equations to describe the operation of the guidance system and writing of computer programs to use the equations to analyze system performance.

c. Analysis of Guidance Scheduling Problem - To make efficient use of a given number of onboard operations (corrections and observations), some analysis of the effect of the number and timing of the observations and corrections is made. This is done to elucidate some of the general principles applicable to lunar guidance and also to avoid overspecifying system requirements because of poor operation scheduling.

d. Variation of Parameters - Some of the important onboard guidance system parameters which are varied to determine their effect on overall system performance include:

- Sensor errors (rms)
- Landmark and horizon uncertainties on earth and moon
- Measurement timing errors
- Number of measurements
- Type of measurements (moon-star, earth-star, etc.)
- Initial trajectory estimation errors
- Velocity correction errors

e. Other Errors - Review of other error sources not included in computer program formulation by extrapolation of results from other studies. These errors include:

- Bias-type errors
- Errors in estimation of the astrodynamic constants

f. Double-Angle Methods - Comparison of the effectiveness of theodolite (double-angle) measurements and sextant (single-angle) methods

g. Ranging - Analysis of the usefulness and feasibility of onboard ranging

h. Comparison with Ground-Tracking - Results obtained in this study are compared with results of analysis of midcourse guidance by ground-tracking.

i. System Requirements - The requirements for an onboard guidance system using arbitrary performance criteria

j. Conclusions and Recommendations - General conclusions and recommendations for a guidance system useful in midcourse guidance are given.

5 LUNAR PARKING ORBIT AND DESCENT

The guidance problem in a ballistic orbit about the moon is similar in several respects to the midcourse guidance problem just discussed. First, guidance operations can be conducted in relatively leisurely fashion compared to guidance during powered landing or rendezvous. In addition, thrusting operations will be brief, so that most of the time the guidance system will be operating open-loop and the navigation and control aspects of the problem can be dealt with separately.

Despite these similarities, the radically different trajectories employed in the Orbital and Midcourse Phases make mandatory the separate analysis of the two phases. The differences between the phases are primarily due to the much shorter ranges involved in orbital guidance and the much more rapidly changing conditions than in the Midcourse Phase. These factors might be expected to influence not only sensor accuracy requirements but also the way in which the navigation measurements are implemented.

The Problem Definition for orbital guidance is similar to that developed in the previous section; i.e., trajectory models and system models (consisting of navigation and control schemes) are developed and an analytical plan is formulated for determining the system requirements. However, it should be pointed out that the analytical plans for the Orbital and Midcourse Phases are not exactly parallel for two reasons: (1) the different geometric and dynamic characteristics of each phase tend to place emphasis on different elements of the guidance problem and (2) in the study program it is considered advantageous to investigate problems which might be common to both phases in only one analysis or the other, thereby avoiding redundancy in the study effort.

5.1 SELECTION OF TRAJECTORY MODEL

In subsection 2.5, it was stated that the expected Apollo mission profile served as a model for trajectory selection for the entire study effort - primarily because of the fact that the guidance operations required for that mission are typical of all guidance operations which might be used on a Manned Lunar Mission. The nominal Apollo flight plan, after descent into a low-altitude lunar orbit, consists of a circular orbit during which navigation data is obtained, followed by the separation of a landing vehicle from the mother

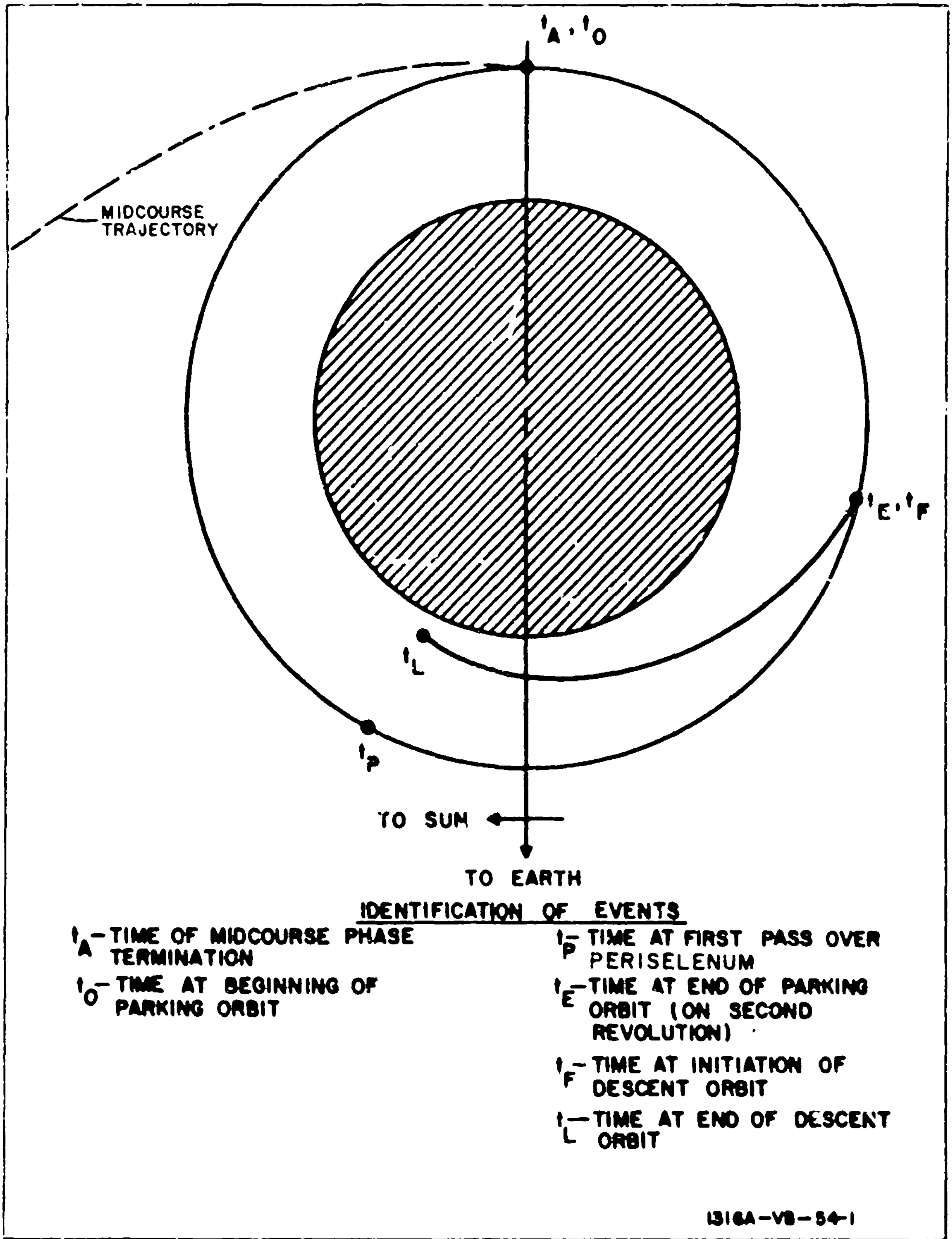

Figure 5-1. Mission Profile for Lunar Orbit and Descent

vehicle. This landing vehicle then fires a retrothrust which places it in the so-called synchronous descent ellipse. This descent (to about a 20-km periselenum) requires about one-quarter of an orbit and the ellipse has an orbital period equal to the period of the mother vehicle, so that if no landing maneuver is initiated, automatic rendezvous will be possible.

Although the mission profile described above was generated for the Apollo mission, there are many reasons for adopting similar plans for any manned lunar landing mission. For instance, use of a parking orbit around the moon has several attractive features:

a. The use of a parking orbit of approximately one revolution allows sufficient time to take navigation data and refine the estimate of position and velocity.

b. The parking orbit affords the opportunity for visual inspection of the landing area from orbital altitude.

c. The use of a parking orbit affords considerable mission safety, since if problems arose in thrusting into a lunar orbit, there would still be a chance of safe return to earth, while such a failure on a direct-descent mission could be disastrous.

The use of the descent ellipse to achieve a 20-km periselenum is advantageous for several reasons:

a. A low-altitude pass over the target site is possible before initiating the powered landing maneuver.

b. A fuel saving will result from starting the powered landing maneuver at as low an altitude as possible (although larger thrust engines are then required).

The particular advantages of the synchronous descent ellipse (as compared to, for example, the 180-degree Hohmann transfer ellipse which is the minimum-energy descent) are:

a. Automatic rendezvous with the mother vehicle is possible if landing is not attempted. (This rendezvous would probably require some maneuvering, but should at least be feasible with reasonable fuel expenditure.)

b. Although the 180-degree Hohmann transfer descent requires less fuel, any desired small plane changes would require another correction about 90 degrees from periselenum. Since the synchronous descent ellipse covers about a 90-degree arc, the plane change could be economically incorporated into the descent velocity pulse so that only one retrothrust need be made.

c. It is expected that the synchronous descent method would be considerably less sensitive to errors in applying the retrothrust due to the shorter arc (90 degrees compared to 180 degrees for the Hohmann transfer). Thus, navigation data during the elliptical descent will not be required.

Other aspects of the mission profile are:

a. It will be assumed that ordinarily no attempt will be made to correct the parking orbit, since exact altitude is not critical and it is more economical to incorporate any desired plane changes in the descent retrothrust.

b. The parking orbit, the descent ellipse, the landing site, and the terminal phase of the midcourse trajectory will all be nominally in one plane. This condition is desirable, of course, in order to minimize fuel requirements.

This completes the discussion of the mission profile for the Orbital Phase. Before going into the discussion of trajectory model generation, however, it should be pointed out that although the Apollo mission profile involves the use of two vehicles, and subsequent rendezvous, there is no reason why the synchronous descent method described cannot be used by a single vehicle. In fact, all the analytical results generated in the study of the Orbital Phase are equally applicable to one-vehicle or two-vehicle operation.

The trajectory model used for this study is simply described: a circular orbit of 200 km is used as a parking orbit and the coplanar descent ellipse is defined by its 20-km periselenum and orbital period equal to that of the parking orbit.

It is assumed that the vehicle would be injected into the parking orbit at a point which is defined by the intersection of the earth-moon centerline and the lunar equator on the far side of the moon. It is further assumed that the desired periselenum of the descent ellipse is above some arbitrary point on the lunar surface as shown on figure 5-2. The figure also illustrates the relationship between the moon, sun, and earth at the time of arrival at the moon. Third-quarter lighting conditions are assumed, as this seems to be in line with current mission planning.

It should be pointed out here that the subsequent analysis is sufficiently general so that none of the factors mentioned in the previous paragraph would be expected to effect any of the analytical results significantly. However, the lighting conditions might influence the types of sensors which could be used for navigation, since it can be seen that for much of the orbit the vehicle will be on the dark or unknown (far) side of the moon.

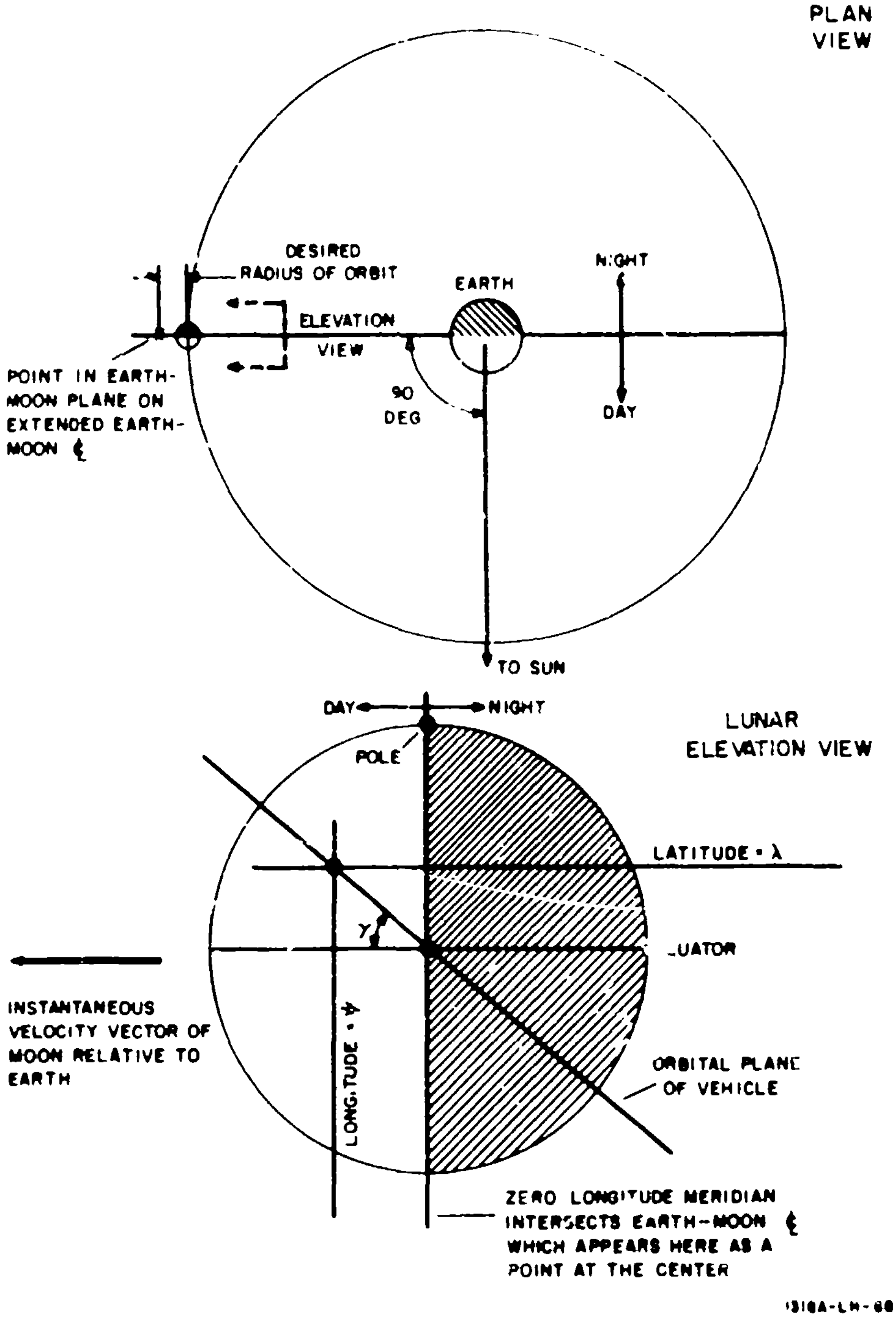

Figure 5-2. Orbital Geometry at Time of Arrival

5.2 SELECTION OF SYSTEM MODEL

The functional block diagram of the general guidance problem in figure 3-1 shows that the description of a guidance system entails specification of the navigation, guidance logic, and control functions. Since this study is primarily concerned with sensor requirements, the major emphasis is on the navigation function and, accordingly, the selection of observables and data processing are discussed extensively in this subsection, while the guidance logic and control functions are considered only to the extent required to generate a reasonable system model.

5.2.1 Navigation Measurements

The analysis of the Orbital Phase deals only with onboard navigation measurements. There are two primary reasons for this choice: (1) there is little published data which deals with the topic of onboard lunar orbital navigation and (2) the use of earth tracking facilities is made difficult by the nonvisibility of the satellite over nearly half an orbit and the fact that all measurements are referenced to the earth rather than the moon.

At the outset, it appears that a wide variety of observables can be measured from the vehicle. These include:

- Time at which a star is eclipsed by the edge of the moon
- Angle between two landmarks
- Angle between a star and a landmark
- Angle between a landmark and either the local vertical or the edge of the moon
- Angle between a star and either the local vertical or the edge of the moon
- Range rate (doppler) measurements with respect to a beacon or to a distinguishable landmark
- Slant range to a beacon or to a distinguishable landmark
- Altitude
- Altitude rate
- Rate of change of local vertical direction

Although the above list indicates a wide choice of observables, not all these quantities can be easily and conveniently measured. Star occulations, angle measurements involving the edge of the lunar disc, altitude, and altitude rate are sensitive to local[1] terrain irregularities and, with the possible exception of altitudes, the effects upon measurements are not easily corrected.

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[1] If a horizon scanner is used to obtain local vertical, measurements may also be affected by departures from a spherical shape but to a lesser degree because of averaging over both time and scanner field.

Measurements involving beacons are possible only when previous missions have landed several accurately located beacons along the orbital track. It is also desirable to avoid any dependence upon accurately known, readily distinguishable lunar landmarks, especially on the unknown or dark side of the moon. In addition to problems of availability, the measurement of landmark-star angles may introduce field-of-view problems; moreover, even with a completely unobstructed view from the vehicle, any particular point on the lunar sphere will be visible only for a limited portion of the orbital duration.

Aside from the above considerations, the information content of the chosen observables must be taken into account. Local vertical rate, for example, does not help to determine the plane of the orbit; the same is true of altitude and altitude rate. The first condition to be satisfied, then, is that the chosen observables must be sufficient to define that orbit completely. At the same time, the number of different observables to be used should be kept at a minimum, for practical reasons. Given these conditions, the selection of parking orbit observables should be governed by the following factors:

- All chosen observables should be readily accessible when needed.
- There should be no loss of information while the vehicle is over the dark or the unknown side of the moon.
- The chosen observables must not place unreasonable requirements (e.g., large amounts of power, a broad field of view, etc.) upon the rest of the onboard system.
- It is desirable that the measurements should lend themselves readily to both manual and automatic operation. This will allow greater standardization between manned and unmanned flights.
- It is desirable that measurements be substantially independent of lunar rotation, and that large amounts of stored astronomical data will not be needed to process the navigation information.

The set of observables consisting of spacecraft altitude (for direct radial information) and measurement of the angles between reference stars and the instantaneous local vertical (for tangential and for out-of-plane information) has the desirable features listed above. This, then, is one set of observables which is selected for analysis. Also, it is obvious that if the altitude measurements are deleted from the above system, the resulting set of observables (local vertical-star angle measurements) meet the listed requirements. Therefore, this system is also analyzed to determine the necessity for altitude measurements in the hopes of achieving an even simpler navigation system.

Briefly, then, the sets of observables considered for analysis are:

a. Altitude and the angle between local vertical and a star
b. Angle between local vertical and a star

No specific method of obtaining local vertical is specified although typical horizon scanner accuracies are used in the error analyses.

5.2.2 Processing of Navigation Data

The problem of determining the orbital path of a spacecraft from on-board observations is quite similar to the trajectory estimation problem in the Midcourse Phase. In either case, the orbit is completely determined by specifying six initial conditions in equation 4-1. Theoretically, this could be accomplished by simultaneously measuring all six components of position and velocity. Again, the practical objections to this simple process are:

a. The measurements cannot be expected to be perfect.

b. Devising an instrument to measure both the position and velocity components simultaneously would be quite difficult.

c. There may be some uncertainty in the astrodynamic constants (earth and lunar gravitational fields) which are used in the mathematical model.

Due to the above difficulties, it is desirable that the navigation data-processing scheme have the following features.

a. The scheme should combine all known information (i.e , navigation measurements, initial estimates, and error estimates) in some statistical fashion in order to obtain an estimate of the spacecraft trajectory that is optimum in some sense.

b. The data processing scheme must be capable of using partial data.

c. The estimation process must be convergent, for a wide range of initial uncertainties.

d. The computational and numerical problems must be amenable to solution on a spacecraft computer.

Requirements a through c in the above list are met by both the Weighted-Least-Squares procedure employed by JPL for spacecraft tracking, (Ref. 5-1) and the Minimum Variance technique as applied to the trajectory estimation by Schmidt, Smith and their associates (Ref. 5-2). In item d, however, the Minimum Variance technique enjoys an advantage, since it is a recursive technique in which only the previous estimate is required to be carried from one observation to another. The Weighted Least-Squares (WLS) procedure requires the utilization of all previous data on each new estimate, so storage requirements are higher. But even worse is that fact that the WLS requires the inversion of a matrix whose dimensions are as large as the number of observations. This last operation has been a problem even with large computers.

As a result of the above considerations, the minimum variance data processing technique described in Ref. 5-2, 5-3, and 5-4 is used in this study, since the computational advantages of this technique are advantageous not only in an onboard mechanization but also in the navigation analysis.

Some features and limitations of the Minimum Variance technique which affect the course of the analysis will now be discussed. One important point is that Kalman's original generation of the recursive equations which yield optimal (minimum variance) solutions to the estimation problem were derived for linear systems. In Ref. 5-2, Kalman's formulation was applied by making the assumptions that (1) the observation deviations (from nominal values) are linearly related to the trajectory deviations and additionally (2) that the trajectory deviations at one time are linearly related to deviations at some other time. Of course, neither of these assumptions is exactly true, but it was assumed that as long as the vehicle was reasonably close to its nominal (trajectory), the approximations are sufficiently accurate.

Another requirement of Kalman's original formulation is that the dynamic models representing both the system and the error processes be known exactly and that, additionally, the statistical properties of the error process be known exactly. Again, neither of these conditions will be met in a real case, so that some degradation from optimum can be expected.

While investigations of some of the above items have appeared in the literature, an effort is made in the analysis to investigate the remaining items, in addition to the determination of sensor requirements. Specific points which are analyzed in these areas are listed in subsection 5.4, the Analytical Plan.

5.2.3 Guidance Logic and Control Functions

As the primary emphasis in this study is on development of sensor requirements, a heavy analytical effort on guidance logic and control requirements is not attempted. Instead, it was decided to choose a simple velocity-correction scheme which yields a reasonable system model for the analysis of navigation requirements.

In this study it is assumed that a high-thrust rocket motor will be used to send the landing vehicle into the descent ellipse; i.e., the descent maneuver is assumed to be an impulsive velocity change.

Guidance logic can conveniently be classified either fixed time of arrival (FTOA) or variable time of arrival (VTOA). The FTOA guidance technique is based on a simple principle: a ballistic trajectory can be uniquely defined by specifying two position vectors and the time between them. Thus, given any initial position at t_o, some desired position at t_E can be achieved by controlling the three components of velocity at t_o; i.e., by application of an impulsive velocity change. This is the guidance logic assumed in this study.

It is recognized that on an actual mission some VTOA scheme will probably be used to conserve fuel and to achieve a horizontal velocity vector at the end of the descent ellipse. However, it is assumed that the use of an FTOA logic would be sufficient to illustrate the effects of error propagation in order to specify sensor requirements.

5.3 ANALYTICAL MODEL

This subsection lists the important features of the trajectory and system models.

Trajectory Model:

Synchronous descent mission profile.
Circular parking orbit at 200-km altitude.
Synchronous descent ellipse to 20-km periselenum.

System Model:

Onboard navigation using the following observables:

a. Altitude and star/local-vertical angles

b. Star/local-vertical angles alone

Minimum Variance data processing technique
Impulsive velocity changes
Fixed-time-of-arrival guidance logic

5.4 ANALYTICAL PLAN

In Volume III sensor requirements for the models chosen are determined by making parametric variations of the sensor accuracy and the number of measurements and comparing the results achieved with some desired error volume. In addition, considerable effort is spent in examining some practical aspects of the application of the minimum variance data-processing technique to the Orbital Phase navigation problem. These topics include:

a. Determination of whether or not the nominal trajectory can be used as a reference for orbital estimation throughout the flight. If not, the estimated trajectory must be used, entailing recalculation of the reference trajectory after each new observation (or after every few observations).

b. Determination of whether or not 2-body equations* can be used. Use of these equations is desirable for computational simplicity.

c. Determination of the effect of large initial errors.

d. Determination of the effect of errors in timing the measurements.

e. Determination of the effect of uncertainty in the error statistics.

To analyze these and other error sources, a computer program is generated which is capable of both Monte Carlo simulations and the covariance matrix analysis employed in Ref. 5-2, 5-3, and 5-4. In this way the effects of nonlinearities (which are not considered in conventional covariance analyses) can be determined, and even the validity of the covariance matrix approach can be established. This approach is a step toward a similar plan, which has been advocated on a grander scale in Ref. 5-5 "The ultimate test of any guidance scheme is a complete launch-to-impact mission simulation, with Monte Carlo selection of all random disturbances that affect the measurements and trajectory coordinates."

* Two-body equations are the equations of motion generated by assuming that the vehicle is moving in the gravitational field of a point mass at the center of the moon.

6. LUNAR LANDING PHASE

The Landing Phase of a lunar mission is defined as that segment of the flight which begins with turnon of the main landing engines at periselenum of the descent ellipse and terminates with descent of the landing vehicle to the lunar surface.

The guidance problem during the Landing Phase is considerably different from midcourse or orbital guidance for several reasons:

a. Since the vehicle is thrusting continuously, the guidance system is operating closed-loop and the navigation, guidance logic, and control aspects of the problem are much more intimately related than in ballistic flight.

b. The short time length of the powered flight (typically 200 to 300 seconds) and the rapidly changing dynamic conditions preclude the use of a highly complex or time-consuming navigation procedure.

c. Knowledge of the landing vehicle's position and velocity with respect to some astroinertial coordinate system may no longer be sufficient because of uncertainties in target location and tighter terminal accuracy requirements. In other words, a target-referenced coordinate system is now desirable.

Despite these differences the general order of this Problem Definition and subsequent Analytical Solution (Volume III) are the same as for the Midcourse and Orbital Phases. One difference in the approach, however, is that in this section, trajectory models are considered after the specification of system models, because for the Landing Phase the system models chosen directly influenced the choice of trajectory models.

Another important point is that for the analysis of the Landing Phase, a two-dimensional analysis is used, primarily to simplify the mathematics, since the equations of motion during powered descent are considerably more complicated than in ballistic flight. This procedure is felt to be justified by the desired end-results (sensor requirements) of the study; i.e., sensor accuracies capable of defining the downrange and vertical state of the vehicle should certainly be sufficient for defining the cross-track uncertainties.

6.1 MISSION PROFILE AND GEOMETRY

There are two methods of accomplishing powered soft landings on the moon. The moon can be approached directly with the Landing Phase immediately following the Midcourse Phase. The second technique is to follow the Midcourse Phase with a parking orbit and then land. The second technique is preferred for manned flight because of crew safety considerations. (It provides increased abort capability and an opportunity for observation before committing the vehicle to a landing.) Since manned lunar flights are given top priority in this study and since the Apollo mission has been selected as a nominal mission profile, it is assumed that the landing phase is initiated at periselenum of a synchronous descent orbit (described in Section 5). Landings both with and without guidance aids on the surface are considered.

The landing operation can be divided into two subphases. The first subphase is the descent from a ballistic trajectory to a state of zero vertical and horizontal velocity relative to the moon at some small altitude above the surface. This zero-velocity condition is referred to as the hover state. The second subphase is the descent from hover to the lunar surface. In this study, only the descent-to-hover subphase was studied, since it was assumed that the hover-to-touchdown subphase would be a manual operation.

The basic landing geometry and several of the parameters to be used during the subsequent discussion and analysis are illustrated in figure 6-1. In this figure the landing site lies in the vehicle plane of motion. X and Y form a cartesian coordinate system fixed to the landing site, while r and θ form a moon-centered polar coordinate system which is also referenced to the landing site.

6.2 FUNCTIONAL DESCRIPTION OF LANDING GUIDANCE SYSTEM

The functional block diagram of figure 6-2 emphasizes the three subsystems considered in this section: Navigation, Guidance Logic, and Control. Figure 6-2 shows that the landing guidance system can be characterized by specification of these three systems and the dynamic equations of motion.

The primary goal of this study is to conduct an objective investigation of the effects of sensor accuracies on the landing system. Therefore the discussion of actual system mechanization is held to a minimum, and the system blocks are defined in terms of equations. For example, the Navigation Subsystem is defined by the navigation equations which relate the vehicle state variables to the navigation observables. In addition, the analytical model of the Control Subsystem is somewhat idealized as will be discussed in paragraph 6.3.3, in order to emphasize sensor requirements rather than attitude control or engine requirements.

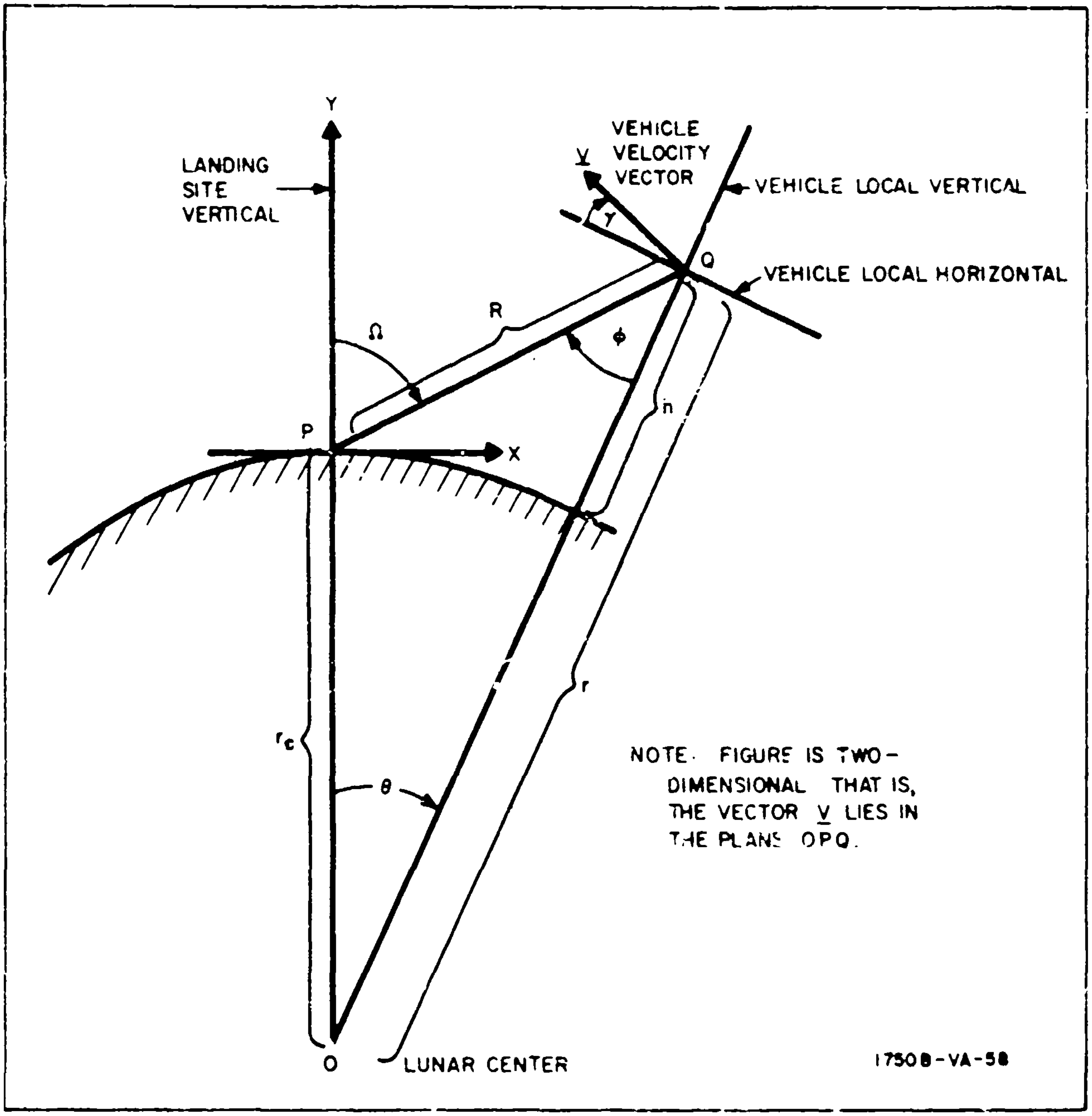

Figure 6-1. Lunar Landing Geometry

The function of the Navigation Subsystem is to acquire and process information which can be used to estimate the vehicle state. The data acquisition instruments are the navigation sensors, and the sensed quantities are termed observables. Since the navigation measurements are subject to error, the actual inputs to the navigation block are the true values of the observables plus some measurement noise.

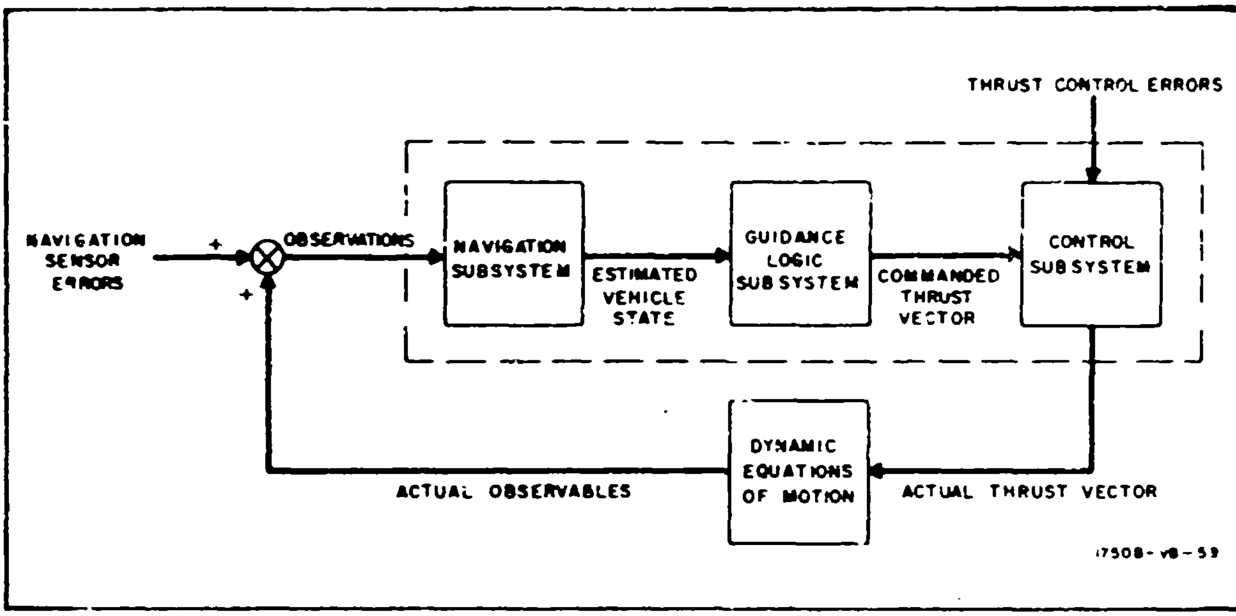

Figure 6-2. General Navigation, Guidance, and Control System Block Diagram

The estimated state variables obtained at the navigation block output form the input to the Guidance Logic Subsystem, whose function is to generate thrust vector commands which will bring the landing vehicle to the desired terminal state. The function of the control block is to implement the guidance commands. As pointed out previously, this function is somewhat idealized in this study in order to concentrate the analytical effort on sensor requirements.

The equations of motion close the loop by relating present and future vehicle state to present and past vehicle accelerations. The general formulation of the equations of motion is the same regardless of the particular vehicle system configuration. However, the actual written statement of these equations is dependent upon the coordinate system in use.

6.3 DESCRIPTION OF SUBSYSTEMS

The following paragraphs discuss the navigation, guidance, and control concepts that are available for use on a lunar landing vehicle. The overall system models to be analyzed in Volume III are synthesized from the concepts presented.

6.3.1 Navigation Subsystem

Navigation refers to the determination of the state of the vehicle. Navigation techniques for powered landing are subdivided into two classes: inertial and direct. Inertial navigation techniques determine position and velocity by using the initial estimates of these quantities and the integrated outputs of accelerometers. Thus, the actual sensed information in this case is acceleration. The principal shortcoming of this approach with regard to the lunar landing problem is that the accuracy of position and velocity estimates can never be better than the accuracy of initial estimates (typically 1000 meters in position and 1 m/sec in velocity as shown in Section 6, Volume III).

The alternative to inertial navigation is the use of direct observation of vehicle state quantities (position, angle, or velocity) or of quantities directly related to the state variables of interest (e.g., measurement of angles to determine position). An advantage of the direct observation approach is that information can be obtained which describes the vehicle state relative to the target or the physical surroundings; e.g., the surface of the moon. This is particularly important with regard to the landing phase because satisfactory control of such quantities as altitude, altitude rate, and displacement from the desired touchdown point are critical to mission success.

For the reasons outlined above, it was decided to base all analysis of navigation during the landing phase on the direct observation of the relative position and motion of the landing vehicle with respect to the lunar surface and the desired landing site.

Consideration is now given to the selection of observable quantities which are the inputs to the Navigation Subsystem and the determination of combinations of observables that provide sufficient navigational information; (i.e., provide sufficient information to allow complete determination of vehicle state in whatever coordinate system is being employed). The first step is to determine that quantities can be observed during a lunar landing trajectory. The available quantities are listed below:

a. Line-of-sight range to a beacon or prominent landmark (R)
b. Line-of-sight range to the lunar surface in some known direction (R_p)
c. Altitude (h)
d. Line-of-sight angle to a beacon or prominent landmark on the surface (ϕ)
e. Direction of local vertical relative to an inertial reference
f. Line-of-sight range rate to a beacon or prominent landmark on the surface ($\dot{R}$)
g. Line-of-sight range rate to an arbitrary point on the surface ($\dot{R}_p$)
h. Rate of change of altitude ($\dot{h}$)
i. Rate of change of line-of-sight angle to a beacon or prominent landmark on the surface ($\dot{\phi}$)

(The above list and the subsequent discussion of choice of observables is based on the two-dimensional geometry illustrated in figure 6-1. However, no loss of generality is thus incurred, since the results are easily extended to cover the three-dimensional case.)

Since there are four state variables for the two-dimensional case (typically, h, θ, V and γ as shown in figure 6-1), a minimum of four different observables are required to determine vehicle state completely (exclusive of local vertical sensing which is used to establish the vehicle coordinate system). Fewer observables could be used as in the Midcourse and Lunar Orbital Phases, but the continuous thrusting and the comparatively brief duration of the entire landing maneuver make this approach of doubtful value during landing.

It is felt that direct observations to determine the vehicle altitude and velocity vector are essential because of the extreme consequences that errors in the knowledge of these quantities can produce. Whether or not observations should be made which will allow determination of the down-range displacement of the vehicle from the landing site is dependent on the mission requirements. If mission success requires that the landing be made accurately at a specific point, determination of θ from observed data is required because this is the only way that unknown initial horizontal deviations can be compensated. The observable quantity in the preceding list that is essential to the determination of θ(or range to the target) is ϕ the line-of-sight angle to a beacon or landmark of known position relative to the desired landing site. If an accurate point landing is not required - i.e., if the mission requirements can be satisfied by a landing anywhere in the neighborhood of the nominal landing site - then observations made solely to determine horizontal position components are not required, because unknown initial horizontal deviations do not endanger mission success if uncorrected during the landing maneuver. In the following discussion, pinpoint landing at a preselected landing site is considered to be a requirement so that continuous determination of state variable θ is essential.

In the determination of what combinations of observables yield complete state information, position and velocity determination are considered separately. Observables yielding position information include range and angle measurements. The only two combinations of observables which give complete position information relative to the landing site are h and ϕ or R and ϕ. Complete velocity information can be obtained from only one combination of two observables; i.e., two measurements of range rate to the surface in nonparallel directions, denoted $\dot{R}_1$ and $\dot{R}_2$. (The direction of each of these measurements must be known with respect to the Landing Phase coordinate system, but this coordinate system is assumed to be established independently.) Other possible combinations providing complete velocity information require knowledge of more than two observables.

However, the extra observations are found to be those required for position determination anyway; e.g., R and $\dot{R}$, ϕ and $\dot{\phi}$. The observables h and $\dot{h}$ can be substituted for line-of-sight range and range rate.

There are several ways to combine the observations required for position and velocity determination given above to form a system providing complete navigation information while using the minimum number of observables:

$R, \dot{R}, \phi, \dot{\phi}$
$h, \phi, \dot{R}_1, \dot{R}_2$
$h, \dot{h}, \phi, \dot{\phi}$
$R, \phi, \dot{R}_1, \dot{R}_2$

Of these, the first two combinations are selected for investigation in this study. Although four schemes are listed, there are really only two basic approaches and these are adequately represented in the first two sets of observables. These two approaches are denoted beacon tracking (observables R, $\dot{R}$, ϕ, $\dot{\phi}$), and doppler navigation (observables h, ϕ, $\dot{R}_1$, and $\dot{R}_2$), the nomenclature being based on a possible velocity determination technique.

As pointed out in the beginning of this section, all these results are directly applicable to a three-dimensional system. Thus, in three dimensions the selected observable sets are R, $\dot{R}$, ϕ_e, $\dot{\phi}_e$, ϕ_a, $\dot{\phi}_a$ (beacon tracking) and h, ϕ_e, ϕ_a, $\dot{R}_1$, $\dot{R}_2$, $\dot{R}_3$ (doppler navigation) where ϕ_e and ϕ_a are the elevation and azimuth components of the sightline angle ϕ.

6.3.2 Guidance Logic Subsystem

The Guidance Logic Subsystem generates acceleration commands based on the estimated vehicle state and sends these to the Control Subsystem (which consists of the landing engine and attitude controls). The obvious goal is to guide the vehicle to the desired endpoint. Fixed guidance techniques such as a scheme employing a predetermined and stored thrust vector program with no provision for modification can be rejected immediately on the basis that they provide no means for removing initial condition errors. Since these errors are likely to be on the order of hundreds or thousands of meters in position and a few meters per second in velocity, unsatisfactory if not disastrous terminal conditions can result. (This same argument is used to reject inertial navigation.) Thus, this study is restricted to the investigation of navigation, guidance, and control concepts using guidance logic which is flexible enough to cope with significant deviations from the nominal initial state.

Guidance concepts for the Landing Phase can be classified conveniently as linear or nonlinear[1] guidance laws. Linear guidance entails the use of linear operations on the observed deviations from some nominal trajectory to derive thrust commands. Nonlinear guidance involves the calculation of the thrust commands from nonlinear operations on the state variables themselves, rather than on deviations of these variables from some nominal values. The primary differences between the two methods are that (1) linear guidance requires the use of nominal trajectory data, whereas nonlinear guidance does not and (2) selection of a particular nonlinear guidance law may limit the class of trajectories which can be flown, whereas linear methods are perfectly applicable to any trajectory.

Since each class of guidance has certain advantages and limitations, both cases are considered in the Landing Phase analysis. In this way, any sensor dependence on guidance logic can be determined. Therefore, a typical guidance method from each of the two classes (linear and nonlinear) is chosen for investigation. The two guidance schemes employed in this study are a linear predictive guidance scheme, hereafter called "linear", and a nonlinear modified proportional navigation scheme hereafter called "MPN." The linear guidance scheme computes acceleration control commands from linear operations on estimated deviations of the vehicle state from the reference state. The basis for comparison between estimated and reference states is the independent variable used in storing the reference information. The independent variable used in this study is time[2].

The linear guidance concept requires that the reference state variables (reference trajectory) be available as functions of time throughout the performance of the landing maneuver. This requirement can be achieved in either of two ways: precomputation and storage or onboard computation of the reference state.

In general, the equations used for linear guidance can involve time-varying coefficients or gain factors. The time-varying nature of these coefficients does not affect the linearity of the system so long as the functions which describe the time-varying coefficients are not also functions of the

[1] Other common terms for the linear guidance method are "implicit" and "delta", and nonlinear methods are often called "explicit."

[2] Other quantities can be used as the independent variable. For instance, in Ref. 6-1 the noise-free performance of a linear system in which velocity is the independent variable is reported.

guidance system inputs which are the observed deviations from the reference state.[3] Time-varying system parameters of this nature must also be either precomputed and stored or computed on board.

The MPN guidance scheme is nonlinear, since the acceleration commands are generated from nonlinear operations performed on guidance system inputs. In the MPN guidance, the estimated state variables are used directly as inputs so that no reference trajectory is required. The MPN guidance concept also employs time-varying system parameters, and these parameters or the functions which determine them are stored on board the spacecraft.

The linear and MPN guidance schemes used in this study were evolved from similar schemes which were shown to give good noise-free results in Refs. 6-1 and 6-3 respectively. Detailed discussion of each guidance scheme is given in Volume III.

6.3.3 Control Subsystem

The function of the Control Subsystem is to convert the commands generated by the Guidance Subsystem into actual vehicle accelerations. In general, satisfactory realization of this function requires control of vehicle attitude as well as engine thrust level and orientation (if gimbaled engines are used). Since detailed investigation of the attitude and engine control function is not considered to be within the scope of this study, this function is not given detailed examination. When in the course of the study it becomes necessary to characterize this control subsystem, it will be described in terms of general mathematical concepts; e.g., time lags, correlation properties, and simple transfer functions. No more detailed consideration is given than is required to produce equations which provide a reasonable approximation to behavior of a typical control subsystem.

6.4 LANDING TRAJECTORY

Landing trajectory analysis and optimization are not considered to be within the scope of this study. However, since typical trajectories are required for the system analysis, several desirable characteristics of lunar descent trajectories are listed below:

Minimum fuel consumption
Vertical approach to the hover point. (This is desirable for three reasons: landing site visibility, terrain clearance, and small horizontal velocities near the end of flight.)

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[3] In Ref. 6-2 the basic requirement for assumption of a linear system is satisfaction of the principle of superposition. This requirement is used to define linearity in this study.

Trajectories which nominally require continuous thrusting at a constant level. (Thus the highest thrust required is minimized and the throttling range over which thrust is varied is small.)

The landing trajectory begins with engine ignition at periselenum of the synchronous descent orbit. Thus, the nominal vertical component of velocity is initially zero. Nominal initial horizontal velocity is determined by initial (periselenum) altitude and the nominal altitude of the circular parking orbit. The optimum initial altitude for the landing maneuver is a function of several factors; e.g., landing site visibility, thrust level, and trajectory constraints. Initial range to the landing site is also a function of several factors. These two quantities (initial altitude and range) define the initial conditions for the powered landing operation.

The nominal termination of the descent subphase analyzed in this study is a zero-velocity hover state directly above the nominal landing site. The hover altitude can be selected arbitrarily and can be set to zero, in which case the hover-to-touchdown subphase is nonexistent. However, the particular hover altitude selected is not a critical parameter in the performance of this study, since the effect of a variation in hover altitude is equivalent to a like variation in the initial altitude. The effect is to translate the entire trajectory upward or downward by the amount of the hover altitude variation.

The following paragraphs discuss the trajectories chosen for analyzing performance of the two guidance techniques (linear and MPN) selected for evaluation.

6.4.1 Trajectory Characteristics Required by Linear Guidance Technique

In the linear guidance method, the guidance commands are determined from estimated deviations from the reference trajectory. No particular constraints on trajectory characteristics are imposed by this guidance concept so that one is free to select the type of trajectory desired.

The nominal trajectory which was selected for analysis of linear guidance is a constant-thrust, gravity-turn trajectory; i.e., the thrust vector is colinear with the velocity vector but of opposite sense. This trajectory model was selected because of the following desirable characteristics:

- Continuous, constant-thrust results in minimum engine size and greater reliability due to the lack of throttling requirements.

- The gravity-turn thrust program yields near-minimum fuel consumption.

- The gravity-turn thrust program offers good terrain clearance and visibility characteristics. Final approach to the hover point is nearly vertical.

- The gravity-turn thrust program automatically rotates the vehicle to the proper attitude at hover (thrusting straight down), so that no last-second, high-velocity rotations are required.

6.4.2 Trajectory Considerations with Respect to MPN Guidance

There is not complete freedom in choosing a trajectory when the nonlinear modified proportional navigation guidance is employed. A class of possible trajectories is generated by varying the gain factors in the guidance equations. Generally the range of allowable values of these factors is limited, and hence the range of trajectory characteristics available with the chosen guidance law is also limited. If the application of a guidance law produces a class of trajectories that are unsatisfactory, the guidance law must be modified.

When a guidance law is found which produces acceptable trajectories, the gain factors are varied within their allowable ranges until the combination producing the most desirable trajectory is found. The nominal trajectory can be generated by applying the resulting guidance law to a mathematical model consisting of the nominal initial state and the assumed vehicle thrust capability.

6.5 SUMMARY OF ANALYTICAL MODELS

Capsule descriptions of the models assumed for the analysis of the Lunar Landing Phase are given below:

Trajectory Models

Nominal initial altitude at 20-kilometers to final altitude of 500 meters. Two trajectory types are:

a. Constant-thrust gravity turn
b. Approximation to optimum fuel trajectory

System Models

Observables

a. Range, range-rate, line-of-sight angle, and angle-rate to reference point (beacon tracking)
b. Altitude, line-of-sight angle, and two components of range rate to lunar surface

Coordinate System

Moon-centered polar coordinates, referenced to target

Guidance Logic

a. Linear predictive guidance

b. Nonlinear modified proportional navigation

6.6 ANALYTICAL APPROACH

The output of the investigation in Volume III is an error analysis evaluating the effects of navigation and control sensor errors on the performance of a lunar landing vehicle. The background analyses required to achieve these results include such items as generation of nominal trajectories, guidance specification (gain factors), determination of navigation equations, and characterization of the control subsystem. The outputs of these preliminary efforts quantitatively define the system to be investigated and are therefore inputs to the error analysis.

Another input to the error analysis is the sensor error model itself. Two general types of sensor errors are considered: bias errors and random or fluctuation errors. The term bias error refers to an error which is random over the ensemble of possible missions but which is fixed for any one member of the ensemble. One can refer to the statistical characteristics of bias errors (i.e., the rms value), but the average is over the ensemble of missions. Random or fluctuation errors on the other hand are random during any single mission as well as over the ensemble of missions.

The analytical assumptions used are listed below.

- The analysis is two-dimensional.

- The gravitational field acting on the landing vehicle is idealized by the assumption of a central force field; i.e., the entire mass of the moon is assumed to be located at the center of the coordinate system and no other forces are considered. The spacecraft's gravitational force on the moon is considered negligible. This approximate astronomical model is considered sufficient for the short-time high-thrust landing trajectory.

- The moon is considered to be stationary during the performance of the descent maneuver. The actual rotational displacement (approximately 1.5 km) of the moon during the time occupied by landing is not negligible. However, the coordinate system used for landing navigation is referenced to a fixed point on the lunar surface. Since this coordinate system moves with the lunar rotation, the relative motion is zero.

There are two basic techniques for performing the actual error analyses. These are simulation of the actual nonlinear navigation, guidance, and control system and simulation of a linearized model of the system. Either approach can be used satisfactorily for the analysis of bias errors, though simulation of the nonlinear system yields more accurate results. There is, however, a definite advantage to linearized analysis for the investigation of random error effects. The reason is that ensemble statistical results can be obtained directly by means of linear analysis, whereas Monte Carlo techniques must be used to evaluate nonlinear systems. Monte Carlo analysis involves making a multitude of simulation runs with different random error records on each run. The result is an ensemble of terminal errors which is analyzed to determine ensemble statistical characteristics. If more than one source of random error is present, the required statistical analysis becomes quite complex.

On the basis of factors considered above, it was decided that bias errors should be evaluated by simulating their effect on the performance of the actual nonlinear system. Random errors, however, are analyzed in terms of ensemble statistical averages and linearized system models.

7. LUNAR ASCENT

7.1 INTRODUCTION

A comparison of typical powered trajectories used to land on the moon and to ascend from the moon to rendezvous indicates a similarity between the two phases in that the time and velocity histories in each phase are almost exactly the same but in reverse sequence. This similarity can be misleading, however, if one assumes that the guidance problem from the two phases is similar. The problems are not similar for the two phases, primarily due to the different end points of each, as will be discussed below.

In the Landing Phase, the landing vehicle must start with some initial estimates of position and velocity (i.e., orbital information) which are referenced to the assumed lunar model and then guided to a target whose coordinates are not known perfectly. Thus uncertainty in target position makes the use of target-referenced measurements and coordinates desirable for fine accuracy. However, in the Ascent Phase both the initial estimate (of the launch site) and the desired ballistic trajectory are referenced to the same astronomical model. In other words, in the Ascent Phase it is assumed that attainment of a particular trajectory is sufficient, since active rendezvous techniques (Section 8) will be used to make up for initial uncertainties as well as guidance inaccuracies in the Ascent Phase. It is assumed that no measurements between the ascending vehicle and the orbiting vehicle will be made, since it is desirable that the ascent guidance be autonomous so that the vehicle may ascend from the surface at any time, if required.

Another unique characteristic of the Ascent Phase is that the initial state estimate (position and velocity of the launch site) must be obtained while the spacecraft is on the lunar surface. Although a more leisurely determination may be allowed, certain accuracy problems arise due to possible anomalies in the moon's composition which cannot be averaged out because the vehicle remains at the same spot. This surface position determination problem is discussed in paragraph 7.5.1.

In the Ascent Phase, it will be assumed that all nominal trajectories are two-dimensional and coplanar. This is the situation which will exist when the launch site is in the plane of the target vehicle's orbit. However, the error analysis itself is three-dimensional, so error components perpendicular to the nominal orbit plane are considered.

7.2 MISSION PROFILES

Three mission profiles were assumed to be applicable to the problem of launching a vehicle from the surface of the moon into a trajectory which is convenient for rendezvous with some orbiting vehicle. These three mission profiles are illustrated in figure 7-1. In figure 7-1(a), the lunar launch vehicle thrusts all the way so that nominally at boost cutoff the positions and velocities of both the launching and orbiting vehicles are identical. In figure 7-1(b) the boost engine is cut off when the launching vehicle position and velocity are such that the resulting ballistic trajectory is on a rendezvous course with the orbiting vehicle. In figure 7-1(c) the launch vehicle is sent into an orbit which is nominally coplanar with the target orbit but at a lower altitude. Transfer of the launching vehicle to the higher orbit is accomplished by reigniting the launch engines at some point L_2. In both figures 7-1(b) and (c), the ballistic phase of the ascent consists of a Hohmann transfer (180-degree) ellipse. Although this particular type of transfer trajectory is not necessarily optimum, it was used in this study to determine the maximum effect of sensor errors.

It should be noted that in figure 7-1(c) the Ascent Phase is independent of the targeting vehicle, so that the results of analysis of this case are directly applicable to the problem of launching a vehicle into a lunar orbit in a non-rendezvous situation.

The following paragraphs discuss some of the important aspects of each type of ascent.

7.2.1 All-Powered Direct Ascent

Rendezvous with an orbiting vehicle can be accomplished by thrusting the launch vehicle continuously until its position and velocity coincide with that of the target vehicle. Some features of this method compared to the other two mission profiles are immediately apparent. First, the weight cost of this method is greatest since, in general, weight in orbit is increased by using a high-thrust, short-duration trajectory rather than a lower thrust and longer time. Second, the time required to achieve a rendezvous is shortest. Finally, this method of achieving rendezvous is expected to be least sensitive to sensor errors, although it is quite sensitive to launch timing errors.

All-powered ascent to rendezvous is not completely unreasonable from a weight standpoint. For instance, this method would be expected to achieve smaller terminal errors than launch to a ballistic trajectory, thus resulting in a possible fuel saving in the terminal rendezvous phase. Thus, although this method of ascent to rendezvous is not necessarily optimum, the sensor requirements for this case are analyzed.

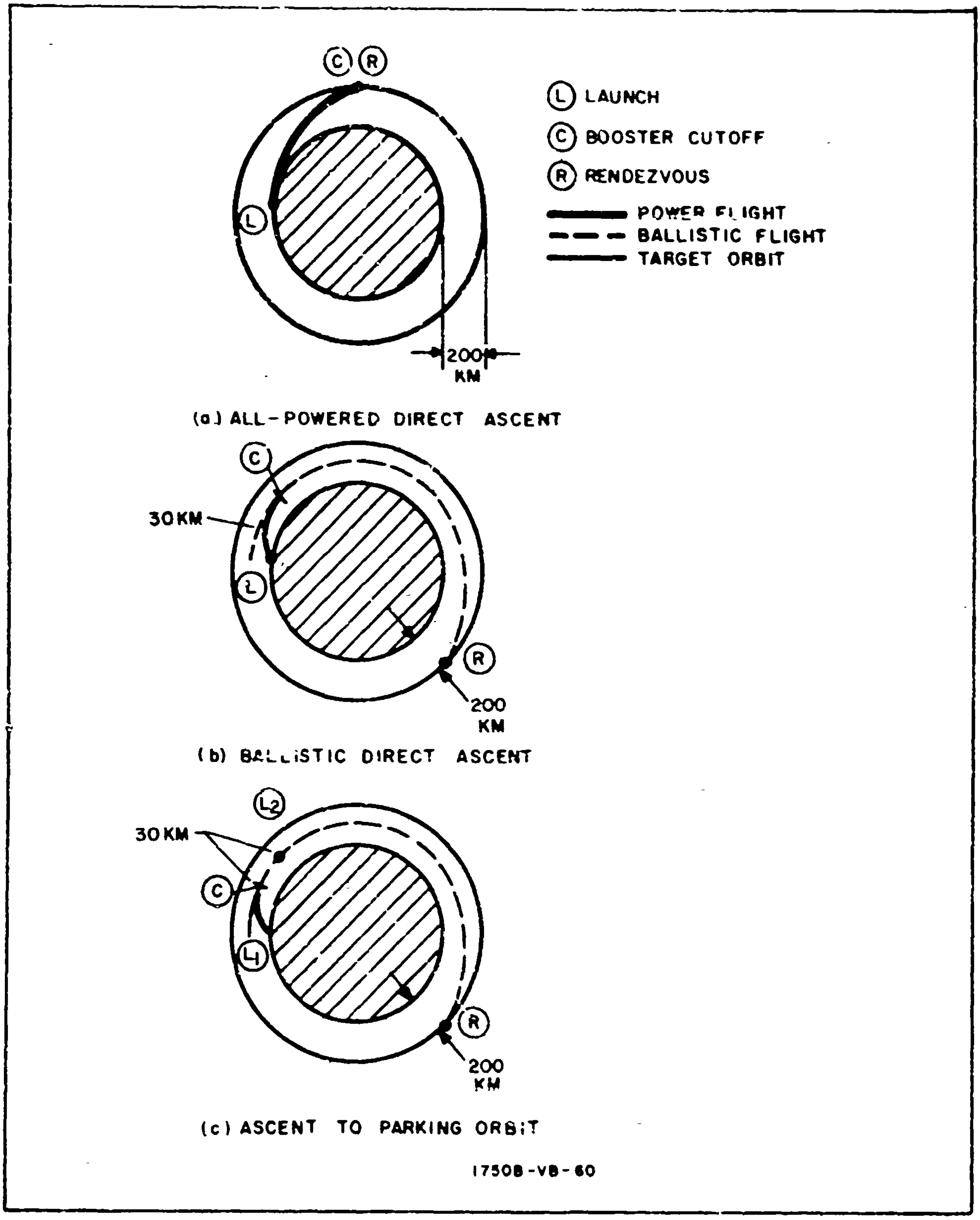

Figure 7-1. Mission Profile for Lunar Ascent

7.2.2 Ballistic Direct Ascent

In the ballistic direct-ascent trajectory assumed in this study, the launch engine cuts off at a 30-kilometer altitude when its velocity is that which will send the vehicle on a Hohmann transfer ellipse (180 degrees) to rendezvous with the target vehicle at 200 kilometers altitude. This trajectory and the all-powered trajectory are at opposite extremes, since the ballistic trajectory is a minimum-energy/maximum-time transfer. However, launch timing requirements are expected to be just as critical and in addition, errors which exist at engine cutoff will propagate over 180 degrees of arc, thus growing larger than errors in the all-powered case.

7.2.3 Ascent to Parking Orbit

In the parking orbit method, the launching vehicle thrusts into a 30-kilometer circular orbit somewhat later than it would for a direct launch. It coasts in this orbit until the central angle difference between the two vehicles is correct for injection into a Hohmann transfer ellipse, at which time the appropriate velocity impulse is added.

The use of an ascent trajectory to a 30-kilometer circular parking orbit obviously relaxes requirements on launch timing, since the different angular rates of the target vehicle (at 200 kilometers) and the launch vehicle will allow time for the two vehicles to achieve the proper angle relationship for rendezvous. However, it should be pointed out that when the launching vehicle is injected from the parking orbit into the transfer trajectory, the timing of this injection is important. Thus, use of a parking orbit does not completely remove the timing requirements for successful rendezvous; rather, it postpones the need for precise timing from surface launch to injection from the parking orbit and makes these timing errors less critical.

7.3 TRAJECTORY GENERATION

In this subsection some of the factors which influence the computation of the nominal trajectories used in this study are discussed.

7.3.1 Trajectory Characteristics

In subsection 7.2 the target orbit is assumed to be in a 200-kilometer circular orbit, and for both the direct ballistic ascent and the ascent to parking orbit booster cutoff occurs at 30 kilometers. The 200-kilometer target orbit which was chosen for consistency with the other phases of the analysis, represents a reasonable figure for a manned lunar mission. The 30-kilometer cutoff altitude was based primarily on safety considerations; i.e., the lower this cutoff can be made, the more efficient is the launch; but obviously there must be a lower limit due to launch and terrain problems. That lower limit was chosen as 30 kilometers.

In both the direct ballistic method and the launch to parking orbit, it is assumed that the transfer trajectory between the 30-kilometer altitude and the 200-kilometer altitude is a Hohmann transfer ellipse. Naturally, this arc could be made shorter, but the 180-degree transfer is one which minimizes fuel at the expense of terminal errors. These errors can be expected to be greatest when boost cutoff errors are propagated over such a long arc. Thus, the assumption of a Hohmann transfer is convenient for analysis, since the complete range of sensor requirements is bounded by the all-powered case as one extreme (maximum energy/minimum time) and the cases involving the Hohmann transfer as the other extreme.

Finally, all trajectories were assumed to be two-dimensional; i.e., the launch site lies in the plane of the target orbit. This is certainly a desirable case from the standpoint of fuel savings, due to the cost of making a plane change. It is felt that this assumption is justified by (1) the primary intent of the study, which is to determine the sensor requirements and (2) the three-dimensional error analysis in which all error components are considered.

7.3.2 Payload Optimization

In all three ascent modes, the nominal powered trajectories generated are approximations of the trajectories which result from an optimum steering program for a constant-thrust vehicle. Assume that it is desired to achieve a circular orbit at some altitude above the lunar surface with some fixed thrust magnitude. Since launch position, launch velocity, thrust, and orbital altitude are all fixed, all that needs definition is the orientation of the thrust vector as a function of time. It has been shown (Ref. 7-1) that the optimum weight in orbit results from use of the so-called "linear tangent" steering program, which is defined by the following equation:

$$\tan \psi = a - bt \qquad (7\text{-}1)$$

where ψ is the angle between the thrust vector and the initial horizontal direction (see figure 7-2), t is time from launch, and a and b are constants determined by optimizing the trajectory for a given set of desired boundary conditions with a particular thrust and burning rate.

The optimum thrust-to-weight ratio is inversely related to the orbital altitude desired (Ref. 7-2). For low-altitude lunar orbits it was found that the powered flight required a total ΔV only about 10 percent greater than the magnitude of the desired orbital velocity. Thus it can be seen that use of a constant thrust program (rather than some on-off or variable thrust program) is reasonable from a fuel standpoint and is desirable for simplicity, both in the analysis and in the actual mechanization.

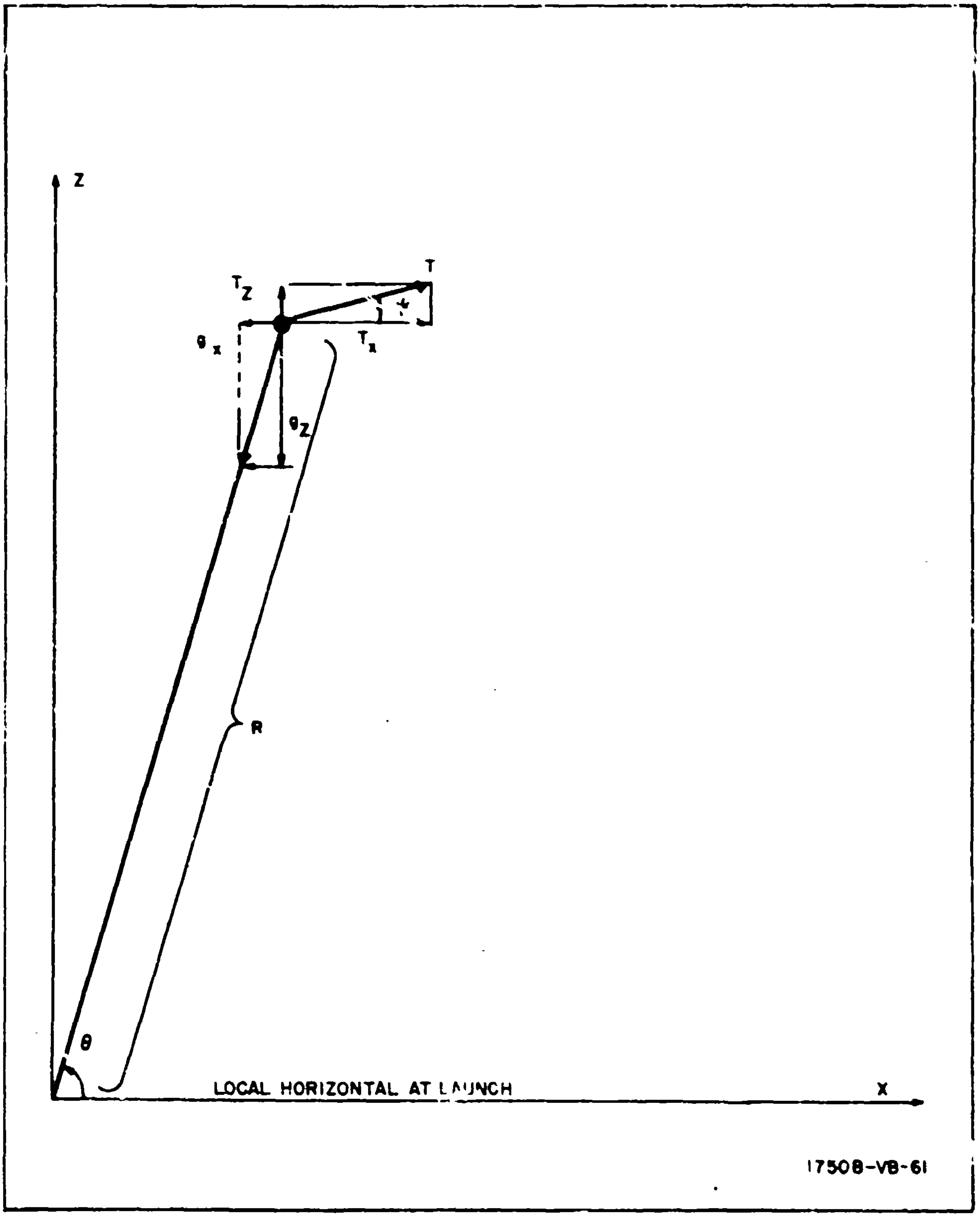

Figure 7-2. Force Diagram of Lunar Ascent Trajectory

7.3.3 Equations of Motion

The equations which describe the motion of a thrusting vehicle in a central-force gravitational field are as follows:

$$\left.\begin{aligned} \ddot{X} &= -\frac{\mu X}{R^3} + \frac{T\cos\psi}{m} \\ \ddot{Z} &= -\frac{\mu Z}{R^3} + \frac{T\sin\psi}{m} \end{aligned}\right\} \qquad (7\text{-}2)$$

where μ is the gravitational constant and the other quantities are illustrated in figure 7-2. Substitution of the steering program (equation 7-1) in equation 7-2 yields the equations of motion for the optimized boost trajectory as shown in Ref. 7-3. However, some approximations have been made in the equations which result in order to obtain trajectories which are similar to the optimized trajectories but for a simpler mathematical form. These approximation methods are discussed in greater detail in Ref. 7-1.

7.4 SYSTEM MODEL

In Section 3, it was shown that a guidance system is completely specified by definition of its Navigation, Guidance Logic, and Control subsystems, where Navigation is the determination of the vehicle state and Guidance Logic and Control are required to alter the vehicle state. The following paragraphs discuss the factors that were considered in selecting the elements of the guidance system model used in this study.

7.4.1 Selection of Observables

The techniques which might be used for guidance of a lunar launching vehicle can be classified according to the observables required as follows:

Guidance Type	Observable
Preset	None
Command	Radioed steering signals
Homing	Range, range-rate, angle or angle-rate from target vehicle
Inertial	Accelerations in inertial system
Celestial	Angle, range and velocity measurements referenced to celestial axes

Of the methods listed above, preset, command, and homing were eliminated from consideration immediately. Preset guidance (i.e., following a preset steering program) is evidently not sufficiently accurate. Command and homing guidance are undesirable since they are not autonomous methods.

The choice between inertial and celestial guidance is a little more difficult to resolve since either method might be sufficient. However, it was felt that since inertial equipment would be on board the vehicle in any case, it would be desirable to use this equipment for navigation and guidance during the Ascent Phase without other observables, if possible. Thus, in the analysis (Volume III) of the Ascent Phase, it was assumed that the sensed inputs to the guidance system during flight consisted of measurements of the accelerations in the three orthogonal directions determined by an inertial reference system and measurements of time. At launch the observables consist of the time and measurement of the direction and magnitude of the gravity vector (paragraph 7.5.2).

7.4.2 Nominal System Configuration

To define a mathematical model of the system for analyzing sensor requirements, it is necessary to be more explicit in defining the guidance subsystems for ascent than was the case for other phases. The system chosen for this study is an inertial system based on a gimbaled stabilized platform. Three single-degree-of-freedom gyros orthogonally oriented on the platform establish a space-fixed reference system. Three linear accelerometers oriented along the reference axes are used to obtain velocity and position within the reference system. The heart of the inertial guidance system is the gimbaled platform with its associated gyros and accelerometers. The platform, gyros and accelerometers constitute the boost sensor equipment.

The navigation and guidance for the lunar ascent is performed in a non-rotating reference frame located at the center of the moon. This reference is assumed to be equivalent to an inertial reference in the analysis, since the orbital motion of the moon during the ascent does not introduce any measurable errors into the guidance system. The orientation of the gyros and accelerometers with respect to the navigational reference frame and the boost trajectory is shown in figure 7-3.

The stabilization of the platform as previously described is performed by three single-degree-of-freedom gyros. The analysis is different if two 2-degree-of-freedom gyros are used. However most recent guidance systems have employed single-degree-of-freedom gyros.

The error analysis (Volume III) is not dependent upon whether or not rate gyros or integrating gyros are used.

As previously stated, a specific type of mechanization must be chosen if an error analysis is to be performed. An alternate method of inertial guidance, the gimballess system, could be chosen. In such a system the accelerometers are not fixed in the navigational coordinate system but have a fixed orientation

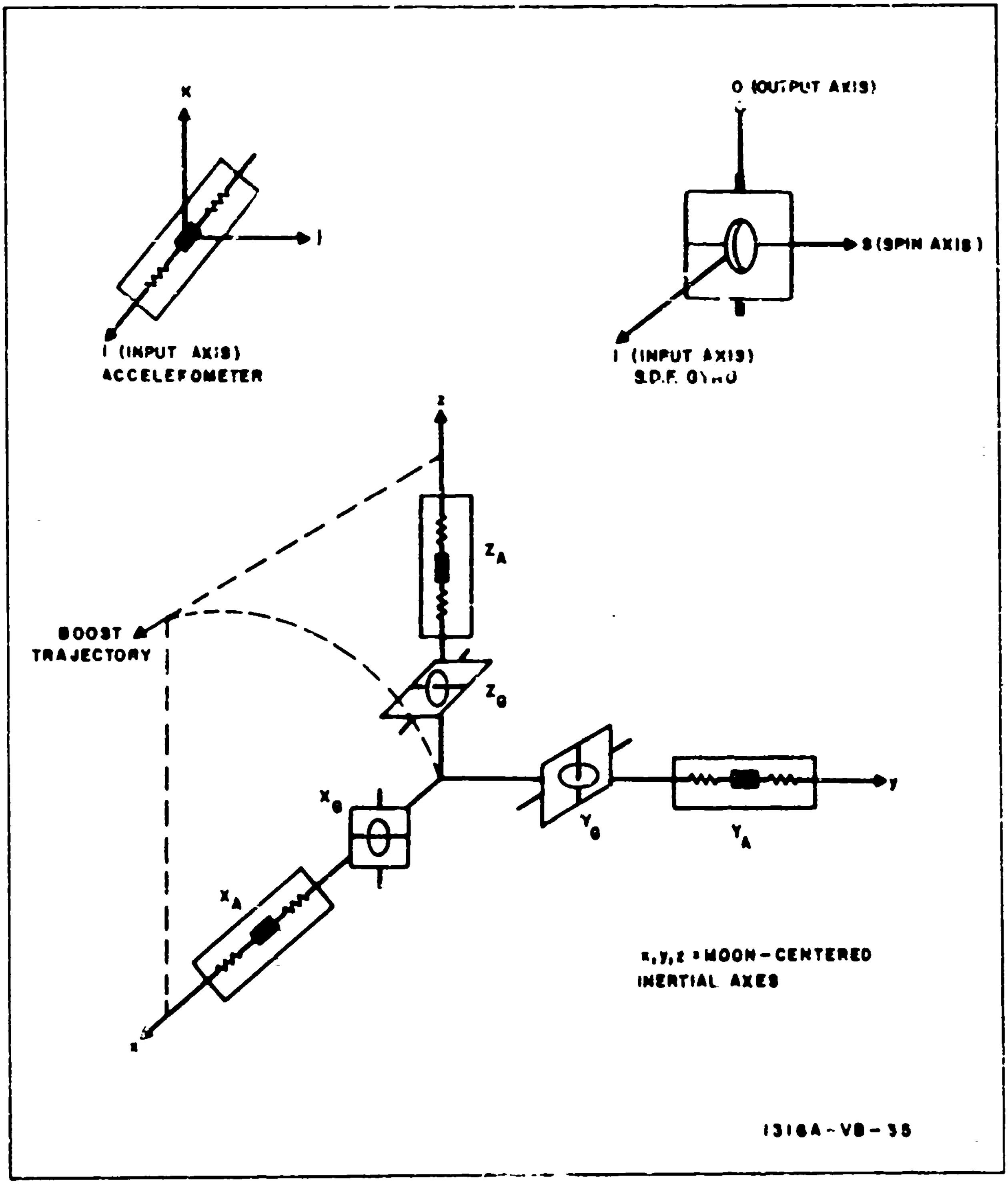

Figure 7-3. Orientation of Gyros and Accelerometers with Respect to Navigation Coordinates

along the axes of the vehicle. To obtain relative position and velocity information, the accelerometer signals must be transformed into the navigation coordinate system. To perform this operation, a transformation matrix which replaces the isolation gimbals of the stable platform must be derived in a computer from knowledge of the attitude rate of the vehicle relative to the navigational reference.

The gimballess system eliminates the weight and errors associated with the stable platform. However, the additional computational requirements dictate a larger and more complex computer. The predominant error sources in the gimballess system are the gyros, since gyro drift and gyro torquing errors can cause the vehicle to depart from the nominal trajectory. The present state of the art of gyro design is such that the errors associated with a gimballess system are comparably larger than with a gimbaled system. In a gimbaled system the gyros operate in very narrow parts of their linear ranges in conjunction with high-gain servo loops. In a gimballess system, rate gyros are normally used. These rate gyros must operate over a much greater range, and as all gyros have a finite linear range, there is considerable difficulty in producing a gimballess system with all-attitude capabilities that has operational errors comparable to those of a gimbaled system. Therefore the gimbaled system was chosen for the lunar ascent analysis.

7.4.3 Guidance Logic

It is not the intent of this study to develop optimum guidance laws for the lunar Ascent Phase but instead to concentrate on determination of sensor requirements. In fact, the error analysis used in this study is not a function of the guidance logic used. However, a brief discussion of applicable guidance concepts is given to indicate the nature of the guidance problem.

As previously mentioned, the position and velocity of a vehicle at thrust termination are needed to determine its resulting ballistic trajectory. These requirements are independent of the type of guidance used. The most generally used technique for missile guidance is based on the "required velocity" concept. The basis of this concept is that at each space-time point along the powered flight path, a required velocity vector can be computed which will make the resulting ballistic trajectory satisfy certain prescribed guidance constraints.

Two of the more common types of guidance equations are based on the required velocity concept. These are explicit guidance equations and delta guidance equations. In explicit guidance, the equations of motion are solved analytically to determine steering commands. In delta guidance, linear perturbations about some nominal trajectory are utilized to generate the steering commands.

The two guidance methods differ in several important practical aspects. Explicit guidance equations are complicated because they are nonlinear, but they require a minimum of precomputation. For delta guidance the reverse is true.

7.4.4 Ascent Rocket Peformance

In the analysis of the Ascent Phase, it is assumed that rocket engine performance is perfect during boost; i.e., there exist no steering errors, time lags or cutoff timing errors. This assumption is made because of the study goals, (i.e., to determine boost sensor requirements, not engine performance requirements). This assumption enables the error analysis to be made independently of the guidance logic used, since engine performance is considered ideal in any case.

7.5 ANALYTICAL APPROACH

7.5.1 Summary of Trajectory Models and System Model

The primary inputs of this volume to the analytical work in Volume III are the mathematical models of the trajectories and systems which are used for analysis of sensor requirements. The features of these models are listed in summary form below:

Nominal Trajectories	System Model
I. Direct powered ascent to 200-km circular orbit.	3 single-degree-of-freedom integrating gyro and 3 linear accelerometers mounted on inertial platform. T/W ratio optimized for desired orbit.
II. Powered ascent to 30-km altitude followed by ballistic Hohmann transfer ellipse to 200-km circular orbit.	
III. a. Powered ascent to 30-km circular parking orbit followed by Hohmann transfer ellipse to 200-km circular orbit after time delay.	
III. b. Same as IIIa except that 150-km parking orbit altitude is used.	

A block diagram of the assumed system model is shown in figure 7-4.

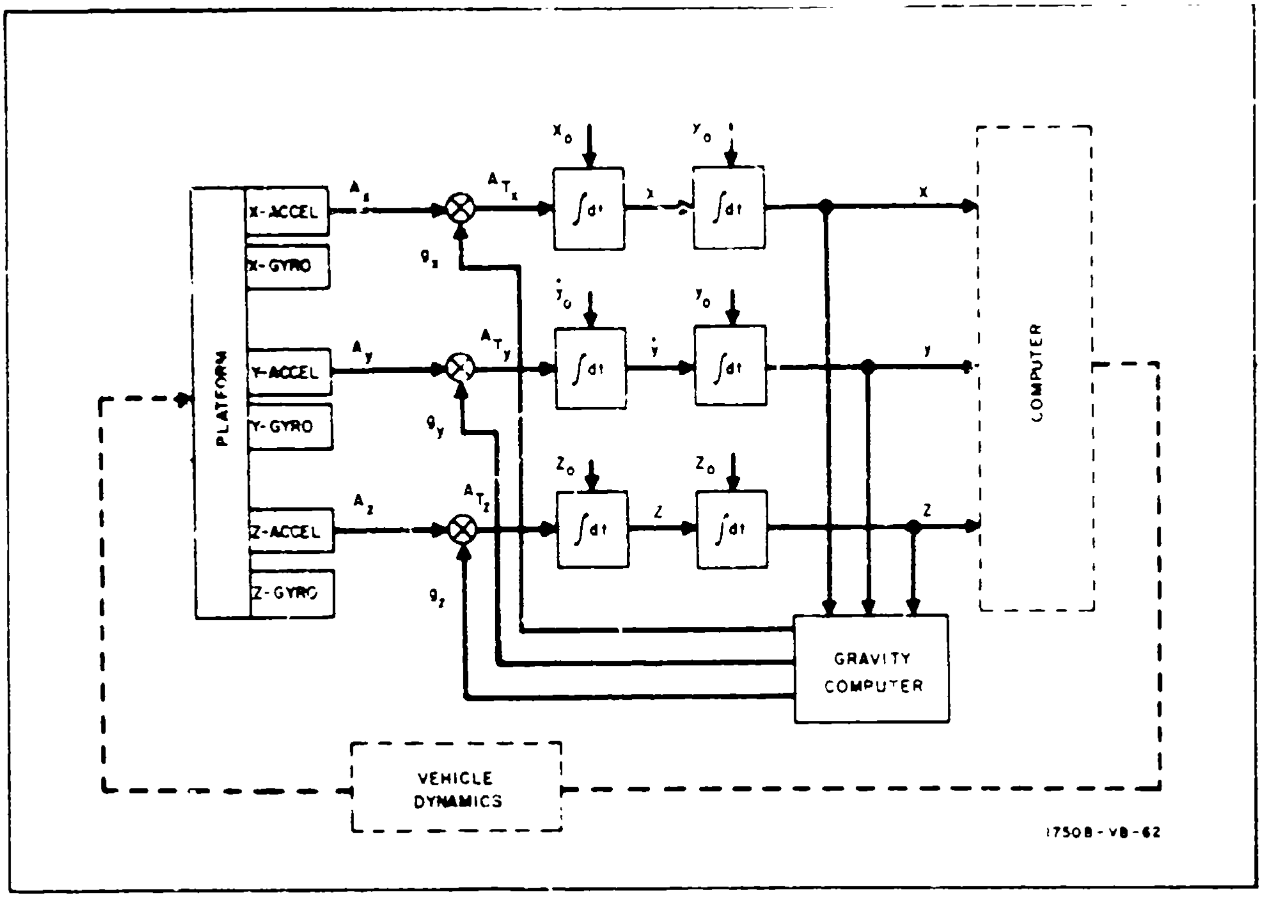

Figure 7-4. Block Diagram of Inertial Guidance Mechanized in Rectangular Coordinates

7.5.2 Launch Site Errors

For an inertial system, the launch site coordinates serve as the initial position estimate. Also, since all information sensed by the inertial devices is referenced to this initial estimate, the accuracy which can be achieved in determining the launch site is of critical importance, since launch site position errors may be a limiting factor on guidance system performance, rather than sensor accuracies. Thus, launch site location accuracy is considered an important part of the analysis of the Ascent Phase.

Ultimately, the problem reduces to the determination of the local vertical and the lunar radius at the launch site. Determination of the lunar radius in turn depends upon measurement of the local gravitational field. For navigation purposes, the local vertical is the radius vector from the origin of the selenocentric coordinate system extending through the launch site. The lunar oblateness, the centripetal acceleration and gravity anomalies cause discrepancies between the plumb line vertical and the local verical as defined above. For navigational purposes, some estimate must be made of the magnitude of the position errors at the launch site. Initial estimates of the lunar oblateness and rotational effects indicate that these two factors may not be significant. The main contribution to deviations in determination of the local vertical will probably be caused by gravity anomalies.

Determination of the magnitude of the radius vector depends on comparing the measured value of gravity at the launch site with the mean lunar gravity. Insight into this problem can be gained by considering a similar situation on the earth. In determining positions of survey points on the earth, it is necessary to consider the points as lying on some mathematical surface such as a sphere or ellipsoid which is taken as representative of the shape of the earth. However, the actual shape of the earth is quite irregular, and only an approximation of its shape can be made. Therefore the first step in defining some representative shape for the earth is to determine the geoid, which is defined as a surface which is everywhere normal to the force of gravity. Ordinarily such a surface coincides with the mean surface of the oceans. Since the maximum value for the acceleration of gravity occurs at the surface of the geoid, the deviation of the magnitude of gravitation from that of the geoid allows computation of the local radius vector. Thus, the geoid is used as a working reference to define a mathematical surface representative of the shape of the earth, and the radius of the earth at any point can be determined by measuring the acceleration of gravity at that point.

The problem of determining location points on the moon is similar, but the difficulties may be greater. The lunar surface has no convenient reference such as an ocean. It is possible to determine the shape of the selenoid (the lunar counterpart of the geoid) from orbiting satellites. From this initial determination, a reference surface can be defined for navigational purposes. Then the navigation problem is greatly simplified if, after a lunar landing, a measurement of the lunar acceleration of gravity can be made and from this measurement a determination of the lunar radius at the landing site can be made.

In this study, the expected error in position coordinates at the launch site is based on similar errors on the earth. This expected error is calculated on the basis of present knowledge of the moon. It is assumed that a lunar reference surface will be determined prior to the time of this mission. The error expectation at the launch site is based on the conservative assumption that the knowledge of the lunar surface and associated lunar parameters at the time of the mission will not be any more extensive than what is known today. It is possible that prior to the lunar mission in question, the surface of the moon including the landing site will have been mapped using high definition photography techniques. Experience on the earth has shown that a high-altitude photograph enlarged to a 1:50,000 ratio will allow positions to be determined to within a few meters. If such were the case, the error in the launch site determination could be sharply reduced.

7.5.3 Error Analysis

The procedure used for determining sensor requirements (in Volume III) consists of an error analysis in which normalized integrals of the thrust acceleration components are used in conjunction with error coefficients for the various sensors. These normalized integrals are time functions of the nominal thrust acceleration components. Since only four trajectories are analyzed, a digital computer program is not required, and results are generated by analytical methods.

At this point, some of the assumptions made in the error analysis are summarized. The nominal trajectory is two-dimensional, but the error analysis made is three-dimensional. It is assumed that the rocket motor performance is perfect; i.e., no steering mechanization errors, control lags or cutoff timing errors occur. Finally, it is assumed that the lunar gravitational field can be represented by a central force field for the analysis if not the actual system. This last assumption is sensible for parametric analysis, since errors will propagate in approximately the same manner in the central force field as in the real physical situation.

8. LUNAR RENDEZVOUS

8.1 INTRODUCTION

The problem formulation for the analysis of Lunar Rendezvous is different from those developed in the previous sections for several reasons:

a. Two vehicles are involved.

b. Rocket operation might be expected to be on-off rather than continuous (as in powered descent and ascent), or short-duration, high-thrust (as for ballistic flight).

c. Although guidance errors in the rendezvous procedure will not necessarily result in disaster, the correct performance of the rendezvous is critical from the standpoint of fuel economy.

In this study it is assumed that the rendezvous maneuvering is done by only one vehicle; thus this vehicle (the "chaser") has an active role while the other vehicle (the "target") is passive.

The analysis done on the rendezvous problem is two-dimensional. This simplifies the analytical formulation of the problem and yet provides results which are valid for three dimensions, since sensor accuracies sufficient for in-plane measurements should prove sufficient for lateral or out-of-plane measurements.

8.2 MISSION PROFILE

8.2.1 Direct and Parking Orbit Modes

The ascent to rendezvous can be performed by direct ascent from the lunar surface or by first ascending to a parking orbit and then thrusting the chaser into a rendezvous trajectory. These two methods are illustrated in figure 8-1.

Of the two methods, use of the parking orbit has the advantage that the different velocities of the two orbits provide time for the chaser to phase

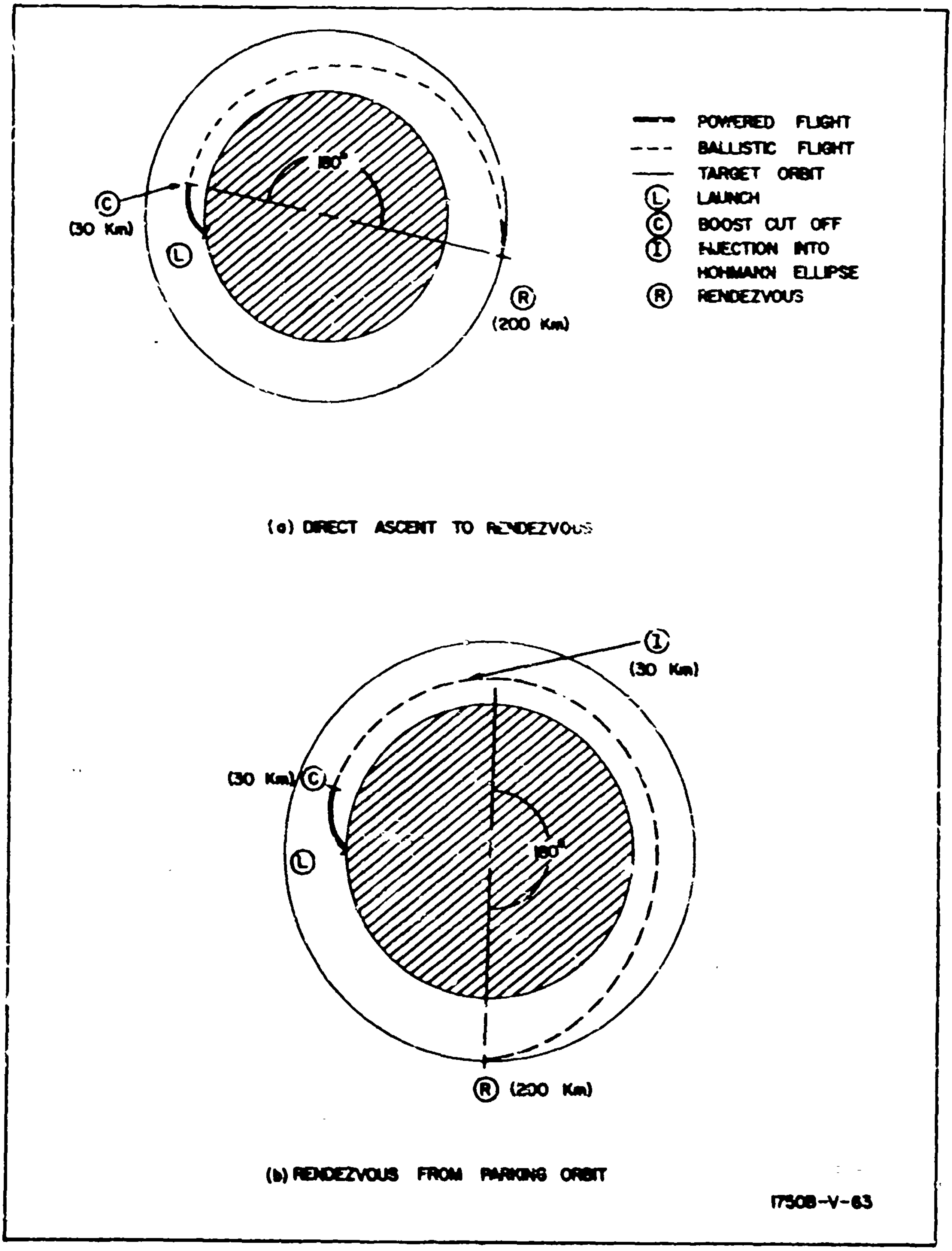

Figure 8-1. Direct Ascent and Parking Orbit Rendezvous

properly with the target vehicle for the rendezvous maneuver, thereby opening up the launch window for ascent from the lunar surface. The use of an intermediate parking orbit is the only rendezvous method considered in this section, since the direct-ascent is treated in the Lunar Ascent Phase (Section 7).

8.2.2 Nominal Geometry and Subphases

The nominal initial rendezvous geometry assumed for this study consists of a target vehicle in a circular orbit at 200-kilometer altitude and a chaser vehicle in a circular orbit at 30 kilometers. The nominal 200-kilometer altitude for the target vehicle is consistent with the rest of the study and can be considered typical for lunar missions. The 30-kilometer parking orbit altitude was considered to be the lowest-altitude orbit which can be achieved safely. (A relatively low-altitude parking orbit is desirable, since its higher velocity allows the chaser vehicle to catch up most rapidly with the target vehicle).

Circular orbits are used for analysis although the actual orbits might be nearly circular ellipses. However, the slight deviations from circular position and velocity will not cause errors to propagate in a radically different manner from error propagation in a circular orbit.

The trajectory traveled by the chaser between the two orbits is referred to as the transfer orbit. In this study, the 180-degree or Hohmann transfer method is used because of fuel considerations; the Hohmann transfer is the minimum-energy two-impulse transfer between orbits. This transfer method is illustrated in figure 8-2. At point I an impulsive change in the horizontal velocity of the chaser is made to send it into an elliptical trajectory having a periselenum of 30 kilometers and an aposelenum of 200 kilometers. At R the tangential velocity is again increased to place the chaser coorbital with the target. However, due to errors in the trajectory injection at I, some active maneuvering by the chaser will be required prior to point R to ensure that the proper terminal conditions (range and range rates) are achieved. This active maneuvering begins at A when the chaser-to-target range has decreased to 25 kilometers.

The rendezvous mission profile shown in figure 8-2 is divided into four sequential subphases: (1) Injection, (2) Midcourse (Coasting), (3) Active Rendezvous, and (4) Docking. A brief discussion of each subphase follows.

8.2.2.1 Injection

The injection maneuver is timed to occur when the chaser vehicle, on the basis of target and/or lunar measurements, is properly phased with the target for the Hohmann transfer rendezvous maneuver. The chaser then imparts the velocity increment ($\cong$ 38 meters per second) required to achieve the transfer orbit.

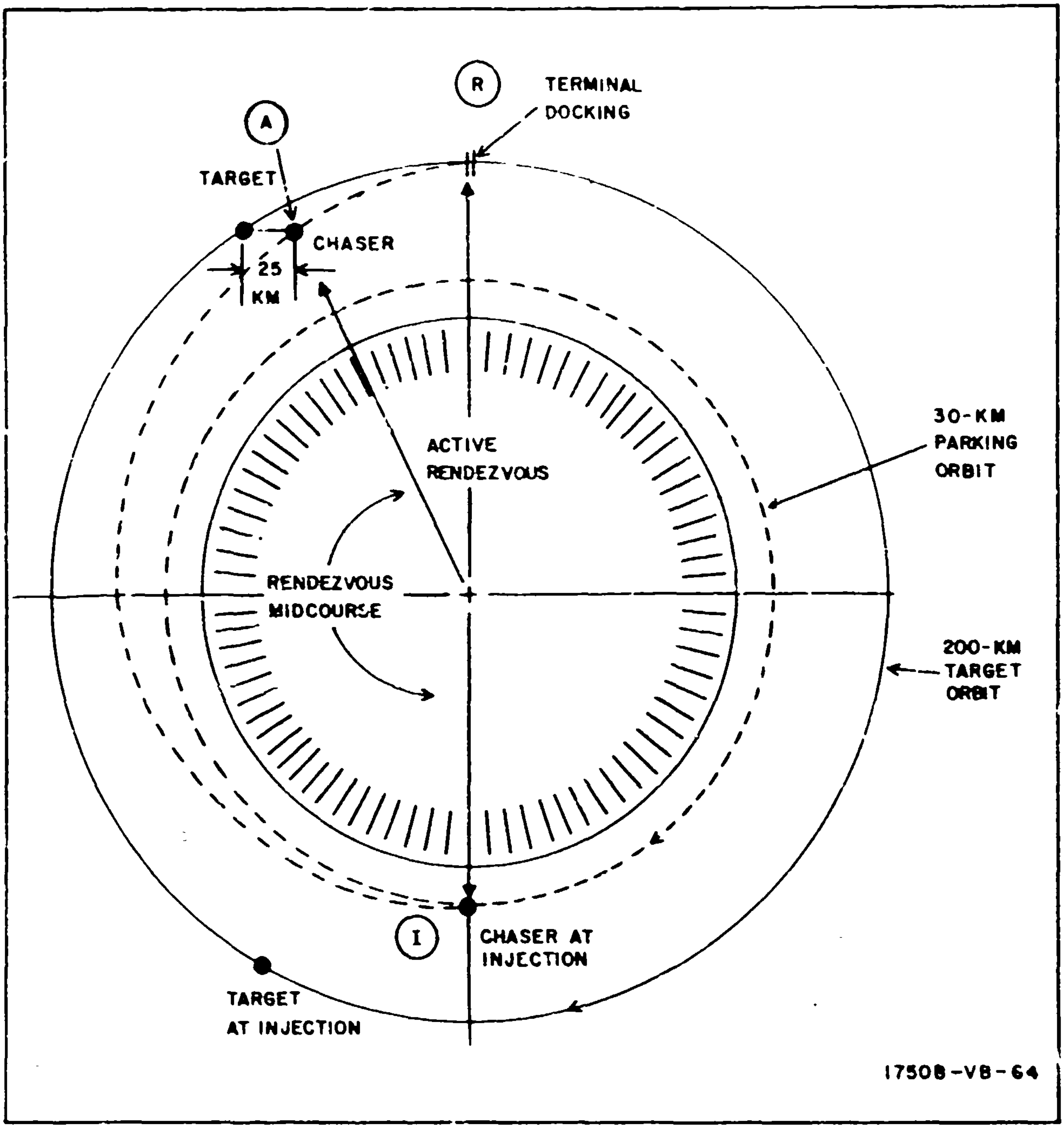

Figure 8-2. Mission Profile for Lunar Rendezvous

8.2.2.2 Midcourse (Coasting)

This ballistic subphase begins immediately after the injection maneuver and continues until active rendezvous maneuvering begins. Injection is assumed to be performed with sufficient accuracy so that no corrective maneuvers are required of the chaser during the midcourse subphase.

8.2.2.3 Active Rendezvous

When the chaser-to-target range has decreased to approximately 25 kilometers, the chaser begins active maneuvering to close with the target in a prescribed manner. The active phase continues until the range has decreased to the assumed standoff range of approximately 50 meters with nearly zero relative closing velocity.

8.2.2.4 Docking

The docking subphase consists of the terminal phase of rendezvous; i.e., the operation of bringing the two vehicles from the standoff range into actual physical contact. In this study it has been assumed that at such short ranges visual observation and manual control are superior to automatic control. Since this study was not specifically concerned with man-in-the-loop guidance operations, the docking subphase was not considered.

8.3 PROBLEM STATEMENT

Control of both injection and active rendezvous is assumed to be from the chaser vehicle. Further, since manned docking is considered beyond the scope of this study and no active control is performed during the midcourse subphase, only the injection and active rendezvous operations are analyzed.

8.3.1 Injection

Since the injection maneuver initiates the rendezvous procedure, any errors occurring in the applied velocity at injection are propagated along the transfer orbit, thereby resulting in deviations or errors near the point of rendezvous. If no errors occurred at injection, there would be no requirement, theoretically, for the active rendezvous phase, since no errors would exist at the nominal rendezvous point and only a single impulse would be required to place the chaser co-orbital with the target.

Observables upon which the injection can be based are:

- Altitude
- Velocity
- Relative range
- Relative range rate
- Attitude (pitch and yaw)
- Central angle
- Timing
- Inclination

Injection errors may develop as a consequence of inaccuracies in measurement of these observables or in execution of the velocity pulse, the resultant errors being introduced in position, velocity vector, and time.

In the analysis of the rendezvous procedure, the navigation requirements for determining the orbits of the chaser and the target prior to injection are not considered but the orbital parameters are assumed known. Therefore, only the effect of the injection errors, rather than their cause, will be considered, in order to determine the level of error which can be tolerated.

8.3.2 Active Rendezvous

The principal area of interest in this study is active rendezvous; i.e., that portion of the mission during which the chaser performs maneuvers to close with the target vehicle to within a short stand-off range (approximately 50 meters) at zero velocity. This maneuvering is necessary because of injection errors which result in conditions that are nonideal for the intended Hohmann transfer.

The primary aim of this study is to obtain sensor requirements. To obtain these requirements, a model system is formulated to determine the performance obtained with different sensor combinations and varying levels of sensor accuracy. Since the Lunar Rendezvous Phase involves closed-loop guidance techniques a guidance system of this type is employed to determine the effect of sensor tolerances. This sytem and the associated control system are defined in the next subsection. The systems are not necessarily optimum for rendezvous but are considered typical and useful for analysis of sensor requirements.

8.4 GUIDANCE SYSTEM MODEL

The system model is considered representative of a rendezvous guidance system. The use of a guidance system such as described in this subsection is not necessarily advocated for use in an actual system; the choice is made to provide an analytical model for determination of sensor requirements.

Complete specification of the guidance system model requires description of the functional blocks shown in figure 3-1. Thus, in the following paragraphs the navigation (observables) guidance logic (control commands) and control (control implementation) functions assumed as a system model are described.

8.4.1 Navigation

8.4.1.1 Observables

The guidance system chosen is a hybrid system in which the sightline rate in the vertical direction is nulled, while a phase-plane relationship of range versus range rate is used to define guidance logic for thrusting in the longitudinal direction.

Because the rendezvous problem is one of determining and controlling the relative (position and velocity) between the two vehicles, observables which yield this information are required. They include the relative range, range rate, line-of-sight angle, and angular rate between the vehicles. Since the study used is two-dimensional, only one component of line-of-sight angle and angular rate is used. In an actual three-dimensional case, however, both an azimuth component and an elevation component of these quantities will be required.

Although no particular method of obtaining the range, range rate and angle rate is specified, the measurements of these quantities are assumed to be contaminated by noise of Gaussian distribution centered about the true value. The contaminated measurements are processed through a digital smoother prior to being used for control purposes. The digital smoothing process is described in detail in Volume III, Section 6.

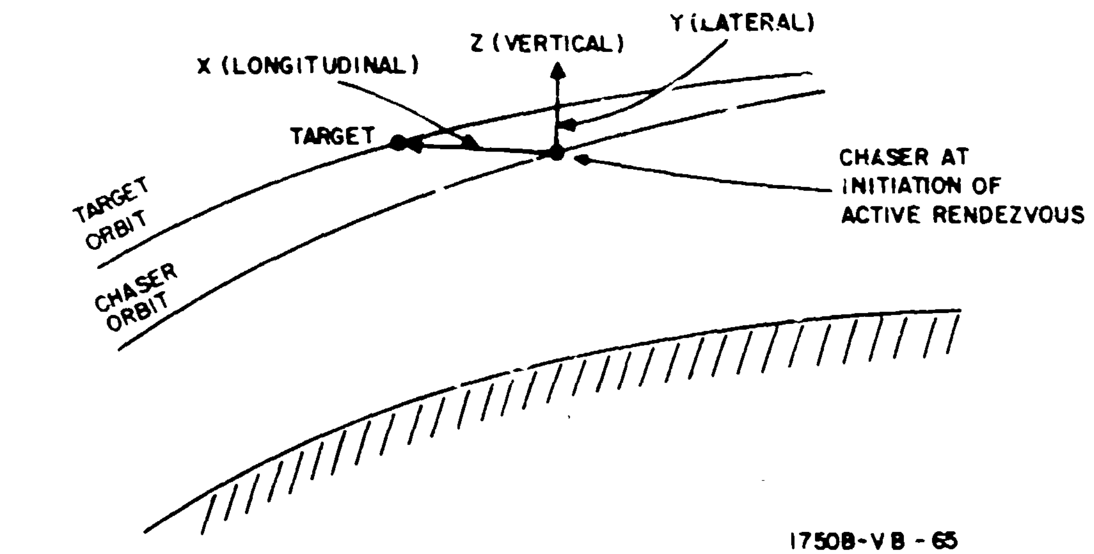

Figure 8-3. Coordinate System for Active Rendezvous

8.4.1.2 Coordinate Systems

The coordinate system used in this study is an inertial system centered at the chaser. This system is locked at time t_I - the time at which active rendezvous begins - with the X axis along the line-of-sight and the Z axis in the general direction of the local vertical. The system is then space-stabilized in this orientation for the duration of the active rendezvous phase and used to define the normal and longitudinal thrusting of the chaser. This procedure is possible because of the relatively constant orientation of the line-of-sight with respect to inertial space during the active rendezvous phase.

8.4.2 Guidance Logic

The guidance logic is different for each of the two coordinate directions (X = longitudinal and Z = vertical) considered in this study. In the vertical direction, thrust commands are generated which will null out line-of-sight rotations. At a distance R, the vertical velocity about the target is given by $R\dot{e}$, where $\dot{e}$ is the line-of-sight. The rocket thrusting time required to null out this error is given by:

$$t_F = \frac{C\,|R\dot{e}|}{|a_z|} \tag{8-1}$$

where a_z is the acceleration (assumed constant) provided by the vertical rocket motor and C= 0.9 to allow for undershoot. In this study, it is assumed that a_z is a bi-level (coarse and vernier) to provide finer control at short range. Other ground rules to inhibit rocket firing due to input noise are also applied.

The guidance logic used in the longitudinal (X) direction is illustrated in figure 8-4, which is a phase-plane plot of range and range-rate along the longitudinal axis. Using the assumed maximum acceleration capability of the vehicle (a_x) as a guide, a pair of parabolic curves, corresponding to constant accelerations less than a_z, can be drawn. These curves represent limits on the combination of range and range-rate, which determine when thrusting should be performed. In figure 8-4, the parabolic curves with a larger negative slope correspond to a more rapid (and more dangerous) closure rate. Use of a lower slope curve corresponds to a safer, more leisurely approach. As in the vertical control mode, a vernier region is provided to allow for fine control.

The equation describing the desired parabolic curve of the phase-plane of the longitudinal axis is as follows:

$$|\dot{R}| > \sqrt{K_i|R - R_f|} \qquad (i = 1,\ 2) \tag{8-2}$$

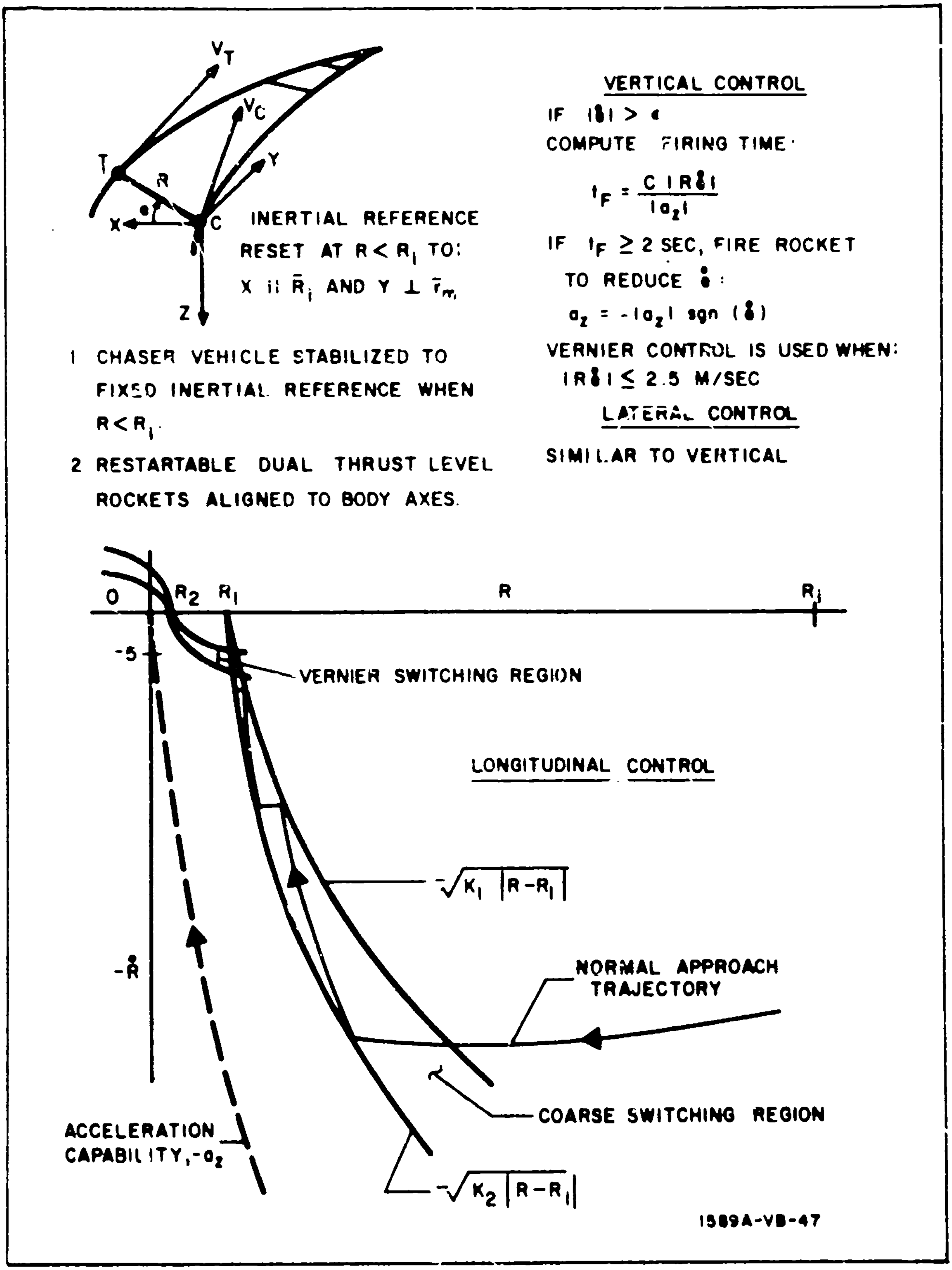

Figure 8-4. Guidance Logic for Lunar Rendezvous

where R is the standoff range and K_i describes the phase-plane slope ($i = 1$ for the lower acceleration boundary; $i = 2$ for the upper acceleration boundary).

When the trajectory intercepts the upper switching boundary, the rocket firing time is computed:

$$t_F = \frac{C\left(-\sqrt{K_2 |R - R_f|} + |\dot{R}|\right)}{|a_x|} \qquad (8\text{-}3)$$

where C = 1.7 for some measure of undercontrol. Thrust is then shut down when the trajectory is driven to the lower acceleration boundary.

8.4.3 Control

The control system assumed in the study is the idealized case of a perfectly accurate rocket engine with no time lags or other errors. Although the results thus obtained are not exact, by eliminating control system errors attention is focussed upon the guidance system sensitivity to sensor requirements and data smoothing, which are of primary concern in the study.

8.5 ANALYTICAL APPROACH

This subsection consists of a capsule description of the trajectory and system models developed in the previous sections and the analytical criteria employed in Volume III.

8.5.1 Analytical Models

The trajectory and system models used in analyzing rendezvous in Volume III are as follows:

Trajectory Model

Chaser initially in 30-kilometer parking orbit uses Hohmann transfer (180 degrees) to attain 200-kilometer orbit of target. Active rendezvous begins at 25-kilometers chaser-to-target range and continues until zero-velocity standoff at 50 meters separation, two-dimensional.

System Model

- Observables - range, range-rate and angle rate
- Data processing - all raw data is digitally smoothed
- Guidance logic - thrust commands to null out line-of-sight vertical component, follow phase-plane parabolas in longitudinal component
- Control equipment - assumed perfect

8.5.2 Analytical Criteria

The goals of the study were to determine sensor accuracies required at rendezvous injection and for the active rendezvous phase. Since the assumption of perfect control makes an error correctable, a propellant consumption criterion was applied to determine the level of sensor errors that can be tolerated.

It is arbitrarily stipulated that no single 3σ injection error shall result in more than a 20-percent increase in the Δv (incremental velocity) over the Δv required for both velocity pulses of the Hohmann transfer when zero errors are imposed on the sensor measurements during active rendezvous. The allowable 3σ level of any given sensor measurement error shall be that which requires no more than a 50-percent increase in Δv over the nominal Hohmann transfer. A given injection error is used on all runs in determining allowable sensor errors, thereby providing a common basis for comparison.

APPENDIX A

EQUATIONS OF MOTION AND ASTRONOMICAL CONSTANTS

The gravitational field acting upon a spacecraft can generally be determined by the following vector equation for the "n-body problem":

$$\ddot{\underline{R}}_{vr} = -\frac{\mu r \underline{R}_{vr}}{|\underline{R}_{vr}|^3} - \sum_{\substack{i=1 \\ i \neq r}}^{n} \mu_i \left[\frac{\underline{R}_{vi}}{|\underline{R}_{vi}|^3} - \frac{\underline{R}_{ri}}{|\underline{R}_{ri}|^3} \right] \tag{1}$$

where $\underline{R}$ is the position vector, μ_i is the gravitational constant of the i^{th} planet, and the subscripts v and r refer to the vehicle and the planet at which the reference coordinate system is located. Thus, specification of the gravitational constants μ_i and the distances between planets completely specifies the gravitational field. The values used as references in this study are listed below:

Earth gravitational constant	$\mu_e = 3.986135(10^{14})\ m^3/sec^2$
Lunar gravitational constant	$\mu_m = 4.8982(10^{12})\ m^3/sec^2$
Sun gravitational constant	$\mu_s = 1.3253(10^{20})\ m^3/sec^2$
Earth-moon distance (assumed constant)	$R_{em} = 382,830$ km
Earth-sun distance (assumed constant)	$R_{es} = 1.4953(10^8)$ km

The formulation of equation 1 assumes that each of the n attracting bodies consists of a spherical, homogeneous massive body. The values used for the radii of the earth and moon are as follows:

Earth radius	$r_e = 6385$ km
Moon radius	$r_m = 1738$ km

Actually, equation 1 is an idealization of the true physical situation in which the attracting bodies are not exactly spherical, nor is their mass uniformly distributed. Thus, the gravitational attraction of real bodies cannot be exactly represented by a point source and equation 1 is not exact.

The effect of an assumed tri-axial figure of the moon was considered only in Section 5, Volume III, which is the analysis of the lunar parking orbit and descent. The equations of motion for this case are derived in Appendix 6 of Volume V.

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